POWER AND ENERGY IMPLICATIONS OF PASSAGE STRUCTURES FOR FISH

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Power and Energy Implications of Passage Structures for Fish

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Abstract.—Fluid mechanic equations are used to show effects of virtual mass force, non-Archimedean buoyant force, and profile drag force on fish in several fish passage structures. Example problems are worked to show computational procedures for calculating net propulsive force, net power, and net energy necessary for fish to swim in a lake, up a steep chute, and through the outlet, barrel, and inlet of a culvert.

Hydraulic Forces Affecting Swimming Fish

Buoyant Force

Fish passage engineers and others responsible for design of fish passage structures have generally assumed that fish surrounded by water are buoyed by a force equal to the weight of the volume of water displaced by the fish, and that the force is directed vertically upward. Thus, weight and buoyant forces appeared to cancel, and both were ignored. Behlke (1987) has shown that this is not the case in fish passage structures where water flows. Fundamental laws of fluid mechanics state that at any point in the fluid, the buoyant force per unit volume of fluid displaced is equal but opposite in direction to the vector gradient of the pressure, ∇p. The buoyant force (B) acting on a fish then is

$$B = (-\nabla p)(\mathrm{Vol}); \tag{1}$$

∇p is the pressure gradient that formerly occurred in the undisturbed fluid at the instantaneous location of the swimming fish's volumetric centroid and Vol is the volume of the fish's body. In a lake, where hydrostatic pressure conditions exist, $\nabla p = \gamma$, the specific weight of water. Thus, the buoyant force would be directed vertically upward and would simply be equal to the weight of the volume of water displaced—the classical Archimedean buoyant force. However, Behlke (1987) showed that if a fish swims up through uniform, steady flow in an open channel that slopes at an angle θ with the horizontal, $-\nabla p$ is reduced from the previous value by a factor $1 - \cos(\theta)$, θ being the slope of the channel. Also, the buoyant force is directed normal to the sloping water surface (Figure 1). Thus,

$$B = \gamma(\mathrm{Vol})\cos(\theta). \tag{2}$$

It should be noted that B is further reduced for fish swimming in aerated water because of a lesser γ.

An interesting example of how B varies in magnitude and direction is that of a free overfall (perched culvert outlet) for subcritical approaching flow. Rouse (1938) showed the pressure distribution in the vicinity of a free overfall for a rectangular channel (Figure 2). If it is assumed that a similar pressure distribution occurs in the longitudinal centerline plane of a culvert at its free overfall, resulting buoyant forces on fish attempting to swim upstream at various locations in the overfall water can be represented as in Figure 3. Clearly, buoyant forces should be considered carefully in design, and great caution must be exercised when buoyant forces acting on fish in passage structures are evaluated.

Profile Drag Force

Traditionally, engineers have been trained to evaluate drag forces on bodies of fixed shape that do not carry their propulsion systems with them. Thus, drag coefficients are determined by laboratory measurements on objects fixed in space so that force measurements are made through the fixation system. Measurements are made at constant fluid velocity. The drag, so measured, is termed profile drag. It is these drag coefficients that adorn engineering fluid mechanics textbooks. However, the development of a system for measurement of drag forces on a body that carries its propulsion system with it is an interesting, challenging, and (to the best of my knowledge) futile exercise.

Biologists have attempted to determine the profile drag on swimming fish by converting measured oxygen inputs to energy inputs (e.g., Brett 1973). However, the assumptions, calculations, and measurements necessary to get from energy input to final net propulsive force, and therefore profile drag force, have not yielded satisfactory results (Webb 1975). Thus, some biologists (and a few mathematicians), well trained in fluid me-

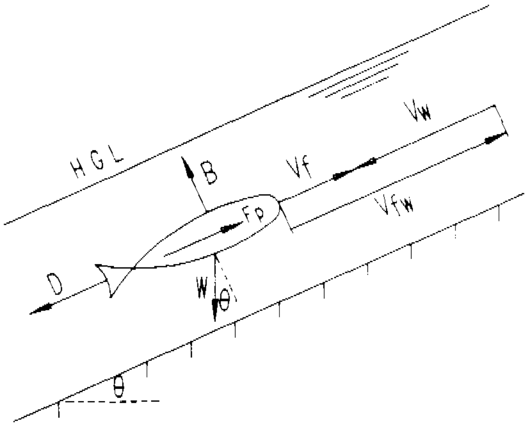

FIGURE 1.—Forces acting on fish swimming in uniform, steady flow in an open channel. B = buoyant force; D = drag; F_p = net propulsive force; HGL = hydraulic grade line; V_f = velocity of the fish; V_w = velocity of the water; V_{fw} = velocity of the fish with respect to the water; W = weight of the fish; θ = angle of the channel with respect to the horizontal.

chanics, have attempted to determine profile drag forces on fish by studying how fish appear to swim and then applying fundamental fluid mechanic concepts to evaluate quantitatively the fluid-generated drag forces on fish. These fluid-dynamicists certainly do not assert that they have the answers, but it is my observation that they, not engineers, are the leaders in this area of research. Thus, I will attempt to use their works to evaluate profile drag and to discuss later the summing of forces that a fish must overcome if it is to move through a fish passage structure.

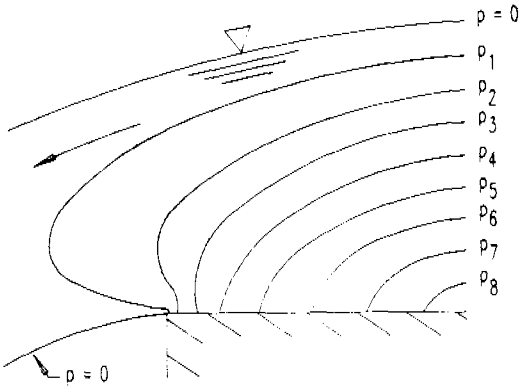

FIGURE 2.—Pressure (p) distribution in vicinity of two-dimensional over-outfall. (After Rouse 1938.)

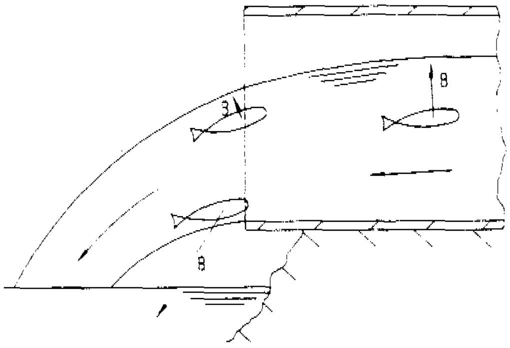

FIGURE 3.—Buoyant force (B) on a hypothetical fish at various locations in the vicinity of a free overfall at the outlet of a culvert.

For an outstanding treatise on swimming hydrodynamics and energetics of fish see Webb (1975). Briefly, however, biologists are presently using as a reference the drag generated by a turbulent boundary layer on a flat plate, the plate being as long as the fish and of sufficient width to have the same wetted area as that of the body of the fish. This is a severe departure from the traditional engineering use of the projected area (A) of a body in the profile drag equation,

$$D = C_d \rho A V^2/2; \tag{3}$$

C_d is the profile drag coefficient, which depends on Reynolds number (N_R) and the shape of the body for which the drag is to be calculated; ρ is the mass density of the fluid; and V is the velocity of the fluid with respect to the body.

Biologists (Webb 1975) have adopted as their standard (and I recommend it to design engineers) the following:

$$D = C_d \rho S V_{fw}^2/2, \tag{4}$$

for which

$$C_d = k(0.072)N_R^{-0.2}. \tag{5}$$

Here, S is the surface area (not projected area) of the fish, V_{fw} is the velocity of the fish with respect to the water, k is a constant that converts the reference drag coefficient to that of swimming fish, and N_R (the fish's Reynolds number) = $V_{fw}L/\nu$, ν being the dynamic viscosity of the water and L the fish's length. Biologists generally believe that the drag on a swimming fish is 3 to 5 times that of the flat-plate reference, so k varies from 3 to 5 (Webb 1975), depending apparently on the fish. Engineers might prefer 0.074 instead of 0.072 as the constant in equation (5); however, this is a minor point that is obscured by other uncertainties of the question of profile drag on swimming fish.

Equations (4) and (5) assume that the fish swims directly into the oncoming current if the water is moving.

If $S = bL^2$ and equations (4) and (5) are combined, profile drag may be expressed as

$$D = bk(0.072)(\rho)(\nu^{0.2})L^{1.8}V_{fw}{}^{1.8}/2. \tag{6}$$

Virtual Mass Force

If an object is accelerated in a fluid, if the fluid surrounding an object accelerates, or if both the object and the fluid accelerate, Newton's second law is operative. Because an object accelerating with respect to its surrounding fluid carries some of the fluid with it, an added mass is accelerated with the object. Thus,

$$F_{vm} = (M + M_a)a; \tag{7}$$

F_{vm} is the force necessary to accelerate the object and the fluid that accelerates with it; M is the mass of the object; M_a is the added mass of the fluid accelerating with the object, and a is the acceleration of the object with respect to the surrounding fluid (Daily and Harleman 1965). The term $(M + M_a)$ is called the virtual mass of the object. In relation to fish, M_a is assumed to be approximately $0.2\,M$ (Webb 1975). Thus, the virtual mass force which a fish feels in an acceleration situation is

$$F_{vm} = 1.2\,Ma_{fw}, \tag{8}$$

a_{fw} being the acceleration of a fish with respect to the surrounding water. F_{vm} is in the direction of a_{fw}.

In one-dimensional motion, the acceleration term a_{fw} of equation (8) is

$$a_{fw} = V_{fw}\partial V_{fw}/\partial s + \partial V_{fw}/\partial t; \tag{9}$$

s is distance measured along the streamline that the fish follows in swimming into the current. Usually, fish passage engineers would not be confronted with unsteady flow, so the final term of equation (9) is ignored. It then becomes necessary to evaluate the simplified form of the right side of the equation. I suggest that the equation be put in the following finite difference form:

$$a_{fw} = V_{fw}\Delta V_{fw}/\Delta s; \tag{10}$$

ΔV_{fw} is the difference in V_{fw} between two points Δs distance apart, and V_{fw} may be approximated by the average V_{fw} over the distance Δs.

The virtual mass force usually acts against forward progress, especially at the outlets of culverts if water surface drawdown occurs there (subcritical approaching flow), at the inlet of culverts, in waterfalls where water is free-falling, and through slots and orifices where water accelerates. Also, in the leaping process fish may accelerate sharply, so during the in-water part of that process F_{vm} is an important force opposing motion.

Acceleration situations, which result in the presence of virtual mass forces, also appear to have an additional effect that hinders passage efforts of fish. Unpublished data (P. W. Webb, University of Michigan–Ann Arbor, personal communication) indicate that water or fish accelerations have some interference effects that elevate profile drag and thus may increase the value of k in equation (6) by a factor of 2 or 3. This is startling information that sounds a loud word of warning to design engineers. In most fish passage facilities the design engineer can control water accelerations in locations where fish must swim if they are to negotiate the structure. If at all possible, provisions should be made for fish to avoid zones of downstream-directed water accelerations.

Weight

Fish may change their volumes slightly, thus changing somewhat their specific weight. This process is generally slow, so it probably has little effect on most fish in passage structures. For fish of similar shape and specific weight, weight (W) is proportional to L^3. Specific weight of fish is usually assumed to be that of water.

Because buoyancy often does not completely cancel weight in fluid dynamic situations found in fish passage facilities, fish weight and buoyancy should always be considered jointly in design until it can be shown that they do cancel.

Other Forces

Yawing, centripetal, and turbulent forces and the effects of mucus on drag reduction are not considered here. I have found little information on these topics that can be converted to design principles. However, as the work of bio-fluid-dynamicists progresses, engineers can expect results that may well be incorporated in future design criteria and procedures.

The principal forces acting on fish in flowing water, and their variation with fish size, are summarized in Table 1.

TABLE 1.—Summary of how forces vary with fish length (L), and fluid and dynamic variables for a species of fish.

Force	Determining fish variable	Fluid or dynamic variable
Buoyant	L^3	$\nabla\rho, \gamma$
Profile drag	$L^{1.8}$	$V_{fw}^{1.8}, \rho, \nu$
Weight	L^3	γ,
Virtual mass	L^3	a_{fw}, ρ

Propulsive Force, Power, and Energy

Propulsive Force

The net propulsive force (F_P) that a fish must generate in order to pass through an element of a fish passage structure is here assumed equal to the sum of the components, in the direction of motion, of the previously discussed forces. However, in some situations the fish must also generate a propulsive component normal to the direction of motion, because of a noncancellation of weight and buoyant force components in that direction (or because of centripetal forces ignored here). That situation is probably most pronounced for fish attempting to enter the mouth of a perched culvert with subcritical flow in the barrel. For a detailed explanation of this and many of the succeeding equations see Behlke (1987) and Behlke et al. (1988).

Some examples follow of net propulsive forces a fish must deliver if it is to move ahead. If it swims upstream in an open channel, culvert barrel, or ramp where uniform, steady flow occurs, profile drag, buoyant, and weight forces impede its forward motion. Behlke (1987) showed that the fish's buoyant force acts normal to the water surface and cancels its weight component normal to the invert but does not cancel the downslope component of weight (Figure 3). Thus,

$$F_P = D + W\sin(\theta), \tag{11}$$

θ being the angle of slope of the channel, water surface, and hydraulic grade line (HGL). If a fish swims through a horizontal, baffled pipe (enclosed flow) or in an open channel inlet or outlet where it swims horizontally but the HGL slopes at angle θ, the buoyant force is greater than the fish's weight and has a downstream component that depends on the slope of the HGL. Here too, B is directed normal to the HGL; thus the propulsive force becomes

$$F_P = D + W\tan(\theta). \tag{12}$$

Where an enclosed (pipe) ladder slopes at an angle ϕ or where a fish follows a streamline at an open channel inlet or outlet that slopes at angle ϕ while the HGL slopes at angle θ,

$$F_P = D + W\{\sin(\phi) + [\cos(\phi)][\tan(\theta - \phi)]\}. \tag{13}$$

A fish attempting to enter a culvert that draws down at its outlet from subcritical flow in the barrel is attempting to move in a zone of accelerating water. If the outlet pool's elevation is sufficient to maintain culvert water depth at or above critical at the outlet lip, approximate hydrostatic conditions exist there, and the HGL slopes approximately at the slope of the water surface. Here the fish is faced also with a virtual mass force in addition to the two forces of equations (12) and (13). Thus, if the streamline (relatively straight) which the fish selects to follow slopes at angle ϕ, then

$$F_P = F_{vm} + D + W\{\sin(\phi) + [\cos(\phi)][\tan(\theta - \phi)]\}. \tag{14}$$

Inspection of this equation and the previous ones that define each of the terms in the right side of the equation (14) reveals that a good deal of information must be measured, calculated, known, or guessed before the equation can be evaluated. W requires a knowledge, usually, of the length of the smallest design fish and its weight–length characteristics. D requires the fish's length, its ratio of surface area to L^2, its velocity with respect to the moving water, and the approximate temperature of the surrounding water. F_{vm} requires the same information as that used to determine W, and it is necessary to know V_{fw} and how it changes through the acceleration zone being studied—for example the vicinity of the outlet or inlet of a culvert.

Power

The instantaneous net propulsive power (Pwr) that a fish delivers to its surroundings is

$$\text{Pwr} = F_P(V_{fw}). \tag{15}$$

Because $V_{fw} = V_w + V_f$, equation (15) can be expressed as

$$\text{Pwr} = F_P(V_w + V_f). \tag{16}$$

Energy

Net energy delivered by a fish in passing through an element of a complex passage structure is

$$E = \int_0^t \text{Pwr}\, dt; \qquad (17)$$

t is the time spent by the fish in traveling through the element. If the fish swims with constant V_f, t equals s/V_f, s being the distance traveled in the element. Thus, if Pwr is described by equation (16) and is not a function of time, equation (17) becomes

$$\begin{aligned} E &= \text{Pwr}\ (V_w + V_f)(s/V_f) \\ &= s\ (\text{Pwr})(1 + V_w/V_f). \qquad (18) \end{aligned}$$

The term V_w/V_f is the price fish pay for swimming. Tilsworth and Travis (1987) reported a 43-min travel time for passage of a single Arctic grayling *Thymallus arcticus* through a culvert of 33.5-m length, so V_f was 0.012 m/s. The water velocity where the fish swam was approximately 0.7 m/s, so $V_w/V_f = 0.7/0.012 = 56.4$. Because it swam and was unable to walk in solid contact with the culvert invert, this fish delivered at least 57.4 times as much energy as it would have if it could have walked through the culvert. (The fish's situation is similar to that of a person who is running to progress slowly upward on an escalator that is moving rapidly downward.)

Equation (18) clearly shows that the faster a fish moves through an element of a structure (fast V_f), the less energy it uses in doing so. On the other hand, it must deliver more power to move quickly through the element. My observations are that fish attempt to get through the most difficult spots in a passage structure as quickly as possible, so they seem to understand equation (18). It is also my observation that if fish cannot see the end in sight, such as in a culvert barrel, they attempt to minimize Pwr. They do so by seeking out the locations where propulsive force (F_P) is minimized, and they reduce (V_f) to some minimum consistent with forward progress, while taking their chances on being able to deliver enough energy to pass through the uncertain element.

Equations (17) and (18) contain the velocity of the fish with respect to a fixed reference (V_f). How fast fish swim in differing situations can only be learned from observations of existing situations. V_f has seldom been recorded when research results have been presented. Inspection of the above two equations clearly reveals how important it is for design engineers to have some knowledge of this parameter if they are to understand why a passage element is good or bad for fish. I suggest that engineers begin to develop catalogs of V_f for different species of fish and different sizes within a species for different difficult fish passage situations. The importance of documenting V_f for different species, sizes, and situations cannot be overemphasized.

Jones et al. (1974), in developing criteria for design of culverts on the MacKenzie Valley highway, assumed that if a fish had the capability of delivering a maximum (for the fish) instantaneous velocity with respect to the water (V_{fw}) of a given value, it actually would deliver that V_{fw} while swimming in a culvert. They then subtracted the estimated water velocity from the instantaneous value of V_{fw} and assumed the difference would be the value of V_f for fish swimming through a design culvert. I question that fish would act as they assumed, though their assumption, if adopted by the fish, would result in a minimal expenditure of energy in passing through long culverts. Fish entering a culvert do not know the culvert length until it is history to them, thus they appear to take power precautions that may or may not bring success in delivering the necessary energy to negotiate the culvert.

Some Biological Implications

The previous equations attempt to present the net propulsive force, power, and energy fish deliver if they are to pass through passage structures. It would appear that each element of the structure should be analyzed and the energy outputs should be summed to determine if the fish is capable of doing the job. However, biological constraints can confuse the efficacy of this summation. For an excellent treatise of the biological (and fluid dynamic) aspects of fish propulsion the reader is referred to Webb (1975). However, a brief, very simplified, overview of the problem is given below.

Modes of Delivery of Energy to Swimming

Fish have two muscle systems for swimming. The red muscle and white muscle systems function quite differently and are capable of delivering vastly different amounts of power and energy, depending on the species and individual fish. Webb (personal communication) described the red and white muscle systems as two different engines in one body.

Red muscle functions aerobically and depends on immediate physiological support systems. The metabolic reactants are generated in small amounts compared with the energy they release and do not accumulate in the tissues. In normal swimming the aerobic, red muscle activity is limited to long-term activity (prolonged and sustained V_{fw}). For many of the fish of interest to engineers, use of the red muscle system results in slow caudal fin movement often of large amplitude. Though this engine that the fish uses for prolonged activity delivers only small amounts of power, it can deliver a great deal of energy over a long period of time. The extent of red muscle in Arctic grayling, as an example, is exceedingly small compared with that of white muscle in the same fish. However, in the Arctic grayling, the red muscle occurs at the outer part of the body next to the skin, where it can deliver a maximum of flutter-bending moment to the caudal fin with a minimum of tension in the contracting muscle.

The white muscle engine in species that engineers are usually interested in accommodating is capable of delivering much more power than is the red muscle. Webb (1975) showed that for cold-water fish the potential power output by white muscle is approximately four times that for red muscle. White muscle functions anaerobically, however, and reactants accumulate in the muscle tissues. This manifests itself by an accumulation of lactic acid in the tissues, a product that diffuses slowly out of the muscles and, through the law of mass action and negative feedback, eventually stops further anaerobic energy production (Eckert et al. 1988). Thus, white muscle activity can only occur for a short time before a long rest is required to eliminate the lactic acid excess in the body. Burst or darting speeds can only be maintained by most species for a few seconds, and if a white muscle energy limit is reached, that muscle cannot soon be used. Negotiating a difficult culvert offers a good example of the importance of red and white muscle activity. If a fish is required to use its limit of white muscle energy to enter a culvert, it might then be able to negotiate the barrel using its red muscle system; but if the culvert inlet presents the need for burst power, the fish probably would not still have enough remaining white muscle capability to negotiate the inlet and would have to fall back downstream.

Example Calculations

Examples of fish energy and power requirements at the two extremes of upstream migration challenges are seen as fish swim through a lake or up a waterfall. Other challenges appear to lie between. Example calculations of power and energy requirements for fish swimming in a lake, a steep chute, and a culvert will be illustrated. For each situation, propulsive force (F_P), net power output (Pwr), and net energy delivered (E) will be calculated.

Lake

Given: $V_w = 0$, $V_f = 1$ m/s at time $t = 0$, $L = 0.5$ m, $b = 0.41$, $a_{fw} = 0.6$ m/s^2, $\nu = 1.55 \times 10^{-6}$ m^2/s, $\rho = 1{,}000$ kg/m^3, $W = 69(\text{N/m}^3) \times L^3 = 8.6$ N, $k = 4$, gravity (g) $= 9.8$ m/s^2.

Determine: Net energy (E) delivered by the fish between times $t = 0$ and $t = 2$ s.

Calculations: Because the HGL does not slope, ∇p is directed vertically upward (equations 1, 2), and B cancels W. Only profile drag (D) and virtual mass (F_m) forces need be considered. Thus,

$$F_P = D + F_{vm} = \text{equation (6)} + \text{equation (7)}$$
$$= bk(0.072)(\rho)(\nu^{0.2})L^{1.8}V_{fw}{}^{1.8}/2 + 1.2Ma_{fw}.$$

Let $C_1 = bk(0.072)(\rho)(\nu^{0.2})(L^{1.8})/2 = 1.17$. Then,

$$F_P = C_1(V_f{}^{1.8}) + 1.2(W/\text{g})(a_{fw}) \qquad (19)$$
$$= C_1(1 + 0.6t)^{1.8} + 1.2(8.6/9.8)(0.6)$$
$$= 4.95 + 0.63 = 5.58 \text{ newtons at } t = 2\ s;$$

$$\text{Pwr} = F_P(V_{fw}) \quad [V_w = 0, \text{ so } V_{fw} = V_f = 1 + 0.6t]$$
$$= 1.17(1 + 0.6t)^{2.8} + 0.63(1 + 0.6t)$$
$$= 10.6 + 1.4 = 12 \text{ watts at } t = 2\ s;$$

$$E = \int_0^2 \text{Pwr}\ dt$$
$$= \int_0^2 [1.17(1 + 0.6t)^{2.8} + 0.63(1 + 0.6t)]dt$$
$$= [1.17(1 + 0.6t)^{3.8}/3.8(0.6)] + 0.63(1 + 0.6t)^2/2(0.6)$$
$$= 11.8 \text{ joules net energy delivered from } t = 0 \text{ to } t = 2\ s.$$

Steep Channel or Chute

Reporting on live-fish experiments with chum salmon *Oncorhynchus keta* in good condition, Orsborn and Powers (1985) gave the following information for two chute studies.

Experiment 1: $L = 0.76$ m, chute length (L_c) $=$ 2.3 m, slope of roughened chute (S_o) $= 0.27$, $V_{fw} = 2.68$ m/s, $V_f = 0.61$ m/s.

Experiment 2: $L = 0.76$ m, $L_c = 2.3$ m, $S_o = 0.36$, $V_{fw} = 2.77$ m/s, $V_f = 0.73$ m/s. Weight and water temperature were not given, so I assume $W = 69L^3 = 30.3$ N and a water temperature of 10°C; thus $\nu = 1.31 \times 10^{-6}$ m^2/s.

Determine: F_P, Pwr, and E for each of the two experiments of Orsborn and Powers.

Calculations: Assume $a_{fw} = 0$, because no acceleration of water or fish was reported. Equation (11) describes this situation. Previous calculations have shown how to calculate D and W, so only the numerical results are shown here.

For the first experiment ($S_o = 0.27$),

$$F_P = D + W\sin(\theta) = 14.5 + 7.9 = 22.4 \text{ N};$$

$$\text{Pwr} = F_P V_{fw} = 22.4(2.68) = 60 \text{ W};$$

$$E = \text{Pwr(length of ramp)}/V_f = 60\ (2.3)/0.61 = 226 \text{ J}.$$

For the second experiment ($S_o = 0.36$), the same sequence of calculations yields

$$F_P = 15.4 + 10.5 = 25.9 \text{ N};$$

$$\text{Pwr} = 25.9\ (2.77) = 71.8 \text{ W};$$

$$E = 71.8\ (2.3)/0.73 = 226 \text{ J}.$$

In the first experiment Orsborn and Powers reported 100% of the chum salmon that attempted the chute successfully negotiated it. In the second experiment, they reported a success rate of only 23%. Because the two values for E above are identical, it is unlikely that the second set of fish were troubled by the net energy that they were capable of delivering. However, the second value for F_P is 16% greater than the first, and the second value for Pwr is 20% greater than the first. Thus, it appears most of the test fish simply could not generate enough power to deliver the propulsive force necessary to climb through the steeper chute. The reported average water velocities down the chute for each of these experiments were virtually identical, 2.07 m/s for the first and 2.04 m/s for the second, so the poor success ratio for the second experiment could not be explained by water velocity and profile drag. The above computations show the $W \sin(\theta)$ term to be 33% greater for the 0.36 slope than for the 0.27 slope, thus illustrating the importance of the fundamental fact that the buoyant force did not cancel the downslope component of the weight force.

Culvert

Hydraulically and as fish passage structures, culverts are very complicated structures, so they command great respect from design engineers. Because fish seek locations of minimum difficulty to swim, average flow velocities in culverts may not be very meaningful except as possible indices to water velocities (V_o) where the fish swim. My experience with Arctic grayling indicates that these fish swim hugging the boundary of culverts, either at the invert or close to the intersection of the water surface with the side of the culvert, whichever is the location of minimum water velocity. In short, in difficult situations that require elevated power outputs (for example, at culvert outlet and inlet), these fish seem to swim with V_f approximately equal to 0.3 m/s almost without regard to V_w so long as their anaerobic limits are not exceeded, though they may swim with much smaller values of V_f in situations of reduced power requirements (e.g., a barrel).

The horizontal angle of skew of water entering a culvert has a profound effect on the horizontal distribution of water velocity in the culvert. My measurements in one such culvert revealed that, along one boundary, water velocities 6 cm from the culvert side and 6 cm beneath the water surface (where fish swam) were only 20% of the average water velocity in the cross section. It appears, from limited observations, that if the angle of skew of water approaching the inlet is 30° to 45°, reduced wall velocity effects may be felt downstream from the culvert inlet a distance of perhaps 8 times the mean water surface width in the culvert barrel. Because this number comes from quite limited data, engineers are encouraged to observe and report their experiences with this extremely important skew effect. Clearly, culvert wall roughness greatly affects the potential for successful passage through the barrel and for success at the outlet and inlet. Multiplate culverts with 5-cm (2-in) corrugations on 15-cm (6-in) wavelengths (Manning $n = 0.035$), or other artificial or gravel–boulder roughness, generate more favorable boundary conditions than do less-roughened culverts. Waves in the culvert resulting from higher water velocities disorient small fish and frequently bounce them from slower water near the culvert wall to higher-velocity regions where they may be swept downstream.

Given: Culvert length (L_c) = 30.5 m, diameter (D) = 3.05 m, Manning $n = 0.036$, stream discharge (Q) at fish passage conditions = 2.27 m^3/s, culvert slope = 0.005. Design fish is Arctic grayling 240 mm in fork length (L_f), and total length of fish (L) is assumed to be $L_f/0.92 = 261$ mm. Downstream scour pool water surface elevation will match the critical depth of flow in the culvert at the outlet for this Q. Assume velocity of fish with respect to the culvert (V_f) for short distance into culvert (0.6 m) at outlet and inlet is 0.3 m/s.

Determine: Power and energy requirements for fish to enter culvert, pass through barrel, and exit the culvert.

Solution for outlet: Calculate normal depth of flow (y_n) and critical depth of flow (y_c) for this culvert and the given conditions. This requires some reference to appropriate charts or some trial-and-error computations that result in $y_n = 0.9$ m and $y_c = 0.63$ m, so the normal velocity $V_n = 1.26$ m/s and the critical velocity $V_c = 2.08$ m/s. Because $y_c < y_n$, this would normally be an outlet control situation. However, because the outlet pool elevation is to match the culvert critical depth at the outlet, the pool elevation forces critical depth to occur at the outlet. If the outlet pool elevation were lower, the critical depth location would move upstream in the culvert, perhaps as much as $4y_c$, so depths farther upstream in the culvert would be somewhat less (and water velocities somewhat greater) than if the critical depth occurred at the outlet. However, the advantages to guaranteeing critical depth at the outlet instead of allowing it to occur further upstream are (1) depths less than y_c and attendant velocities greater than V_c do not occur in the culvert, and (2) the water surface profile drawdown slope and extent at the outlet is controlled, thereby lessening the magnitude of undesirable buoyant effects, and by virtue of smaller water accelerations in the vicinity of the outlet, virtual mass effects are also reduced.

Because of the stated downstream pool elevation and $y_c < y_n$, there exists a hydraulic M-2 water surface profile (Henderson 1966) extending upstream from the outlet. Some quick backwater computations beginning close to the outlet and extending 0.6 m upstream indicate the water surface rise from the outlet to this point is approximately 0.079 m. Thus the slope of the HGL in this zone is approximately $0.079/0.6 = 0.13$, so $\theta = \tan^{-1}(0.13) = 7.4°$. The average cross-sectional water velocity at the outlet is the critical velocity of 2.08 m/s, and 0.6 m upstream it is 1.72 m/s. It is assumed water velocities (V_o) where the fish swim average half of the average cross-sectional velocities. Thus, from equation (10) the average value of the water convective acceleration that the fish is subjected to is

$$\begin{aligned} a_{fw} &= [V_f + (V_{o-\text{outlet}} + V_{o-0.6\text{ m}})/2] \\ &\quad \cdot [(V_{o-\text{outlet}} - V_{o-0.6\text{ m}})/0.6] \qquad (20) \\ &= \{0.30 + [0.5(2.08) + 0.5(1.72)]/2\} \\ &\quad \cdot \{[0.5(2.08) - 0.5(1.72)]/0.6\} \\ &= 0.375 \text{ m/s}^2. \end{aligned}$$

So, $F_{vm} = 1.2(W/\text{g})(a_{fw}) = 1.2(69\,L^3/9.8)(0.343) = 0.04$ N.

Because this short zone at the outlet provides brief, but possibly critical, exposure to the fish, it appears prudent in calculating D to assume the maximum value of V_o (that at the outlet) occurs through the 0.6-m zone. Use equation (6), assume $b = 0.42$, $k = 4$, and water temperature = 4°C, and for V_{fw} substitute $V_o + V_f = 0.5V_c + V_f = 0.5(2.08) + 0.3 = 1.34$ m/s; then profile drag $D = 0.63$ N. Equation (14) for $\phi = 0$, becomes

$$\begin{aligned} F_P &= F_{vm} + D + W \tan(\theta) \qquad (21) \\ &= 0.04 + 0.63 + 0.16 = 0.83 \text{ N}. \end{aligned}$$

Then,

$$\begin{aligned} \text{Pwr} &= F_P(V_{fw}) = F_P(V_o + V_f) \\ &= 0.83(1.34) = 1.1 \text{ W}; \end{aligned}$$

$$\begin{aligned} E &= \text{Pwr} \times \text{time to move through the outlet zone} \\ &= 1.1(2) = 2.2 \text{ J}. \end{aligned}$$

Values of $F_P = 3.1$ N, Pwr = 6.9 W, and $E = 13.8$ J have been reported by Behlke et al. (1988) for similar Arctic grayling entering a culvert through which they passed successfully, so the calculated values would offer no problem to the given fish entering this culvert.

Solution for barrel: Water accelerations in the main body of the barrel are very small upstream from the outlet zone, so equation (11) can be used for F_P. Quick calculations show that for this very flat slope, $W\sin(\theta)$ is insignificant, so it will be ignored, thus $F_P = D$. Here V_o will be assumed to be 0.4 V_{ave} at any s in the barrel, but any assumption for V_o/V_{ave} would have to rest on the engineer's experience with similar existing culverts on other streams and on the body depth of the fish to be passed through the culvert (the bodies of larger fish extend farther from the wall into areas of higher water velocity). For an assumed depth of y_c at the outlet, the water depths and average cross-sectional velocities (V_{ave}) are as shown in Table 2.

In a previous study of Arctic grayling in a single culvert (L_c = 33.5 m), Behlke et al. (1988) found the average value of V_f through the culvert correlated only with fork length of the fish and did not appear to correlate with any other variable. The relationship was $V_f = 11.7L_f - 0.017$ (m/s), L_f being the fork length of the fish (m); for L_f = 0.24 m, V_f = 0.022 m/s. This may appear to be quite slow, but my experience with these fish is that they often move slowly upstream while swimming through a culvert barrel.

The fish begins its journey through the culvert barrel at s = 0.6 m where it exits the outlet zone, and the inlet zone for the fish will begin 0.5 m downstream from the inlet. Because V_o is 30% greater at s = 0.6 m than at 30 m, average values for Pwr will be calculated for s = 0.6 m to 6 m and for s = 6 m to 30 m, and E will be calculated separately for each of these two reaches of the culvert barrel. Average V_o for s = 0.6 m to 6 m is 0.65 m/s; for s = 6 m to 30 m, average V_o = 0.57 m/s. From equations (6), (11), (16), and (18), and for $V_{fw} = V_{o-ave}$ + 0.022 m/s = 0.672 m/s from s = 0.6 m to 6 m,

$$F_P = D = 0.18\ \text{N};$$

$$\text{Pwr} = D(V_{fw}) = 0.18(0.672) = 0.12\ \text{W};$$

$$E = \Delta s(\text{Pwr})(1 + V_o/V_f)$$
$$= 5.4\ (0.18)(1 + 0.65/0.022) = 29.7\ \text{J}.$$

The second term in the final parenthetical term in the energy equation is the price the fish pays for swimming in a moving fluid, and its value (0.65/0.022 = 29.5) compared with 1 is that price. Similar computations for s = 6 m to 30 m yield F_P = 0.144 N, Pwr = 0.085 W, and E = 54.9 J. The total energy used in the barrel is the sum of the two E values, 84.6 J. Field observations and subsequent computations by Behlke et al. (1988) indicate the values for F_P, Pwr, and E computed here are safe for the design fish.

Solution for inlet: At a sharp-edged culvert entrance, streamlines are contracted and the contracted cross section of high-velocity flow in the center of the culvert leaves low-velocity flow, often with upstream velocities, next to the side walls of the culvert. Thus at the inlet end of the barrel, just before they exit the culvert, fish can usually find a rest area in which they may survey the situation ahead. They need not enter higher velocity flow and the entrance drawdown, which may slope sharply, until they are prepared to do so. I doubt that they remain here long enough to recharge their white muscle engine, but they could.

TABLE 2.—Backwater computations for a culvert. D = 3.05 m, n = 0.036, slope = 0.005, Q = 2.27 m^3/s, and s is measured from the outlet lip.

s (m)	Depth (m)	V_{ave} (m/s)	V_o (m/s)
0.0	0.63	2.08	0.83
0.3	0.71	1.76	0.70
0.6	0.72	1.73	0.69
1.0	0.73	1.69	0.68
1.5	0.74	1.66	0.66
3.0	0.76	1.60	0.64
6.0	0.79	1.52	0.61
10.0	0.81	1.46	0.58
20.0	0.84	1.38	0.55
30.0	0.86	1.33	0.53

Because of the flow contraction due to sharp-edged entrance geometry and because the cross section of flow in the culvert is usually smaller than that of the approaching stream, $V_w^2/2g$ must increase as the water enters the culvert, so the water surface must drop at the inlet by an amount equal to the sum of the increased $V_w^2/2g$ and the entrance loss due to initial acceleration and subsequent deceleration of water entering the culvert. The entrance loss is usually expressed as $K_e(V_w^2/2g)$, K_e being a loss coefficient that depends on geometry and V_w the water velocity for the flow just downstream from entrance contraction–expansion effects. Norman et al. (1985) gave K_e = 0.9 for sharp-edged culverts under outlet control. Thus the water surface at the entrance must drop by an amount $(1 + K_e)\ V_w^2/2g$. I assume this occurs in the first 0.5 m of the inlet end of the culvert. Based on the velocity of flow for s = 30 m from Table 2, the entrance drop in water surface is $1.9\,(1.33^2/2g)$ = 0.17 m. This drop occurs principally in the 0.5-m inlet zone, and the slope of the water surface here is 0.17/0.5 = 0.34. Average water velocities against which the fish swims in the inlet zone are assumed to be the same as those of the culvert downstream from the contraction–expansion zone, i.e., 1.33 m/s, because the fish does not have to brave the fully contracted water velocities. Because this is a zone of short, high-power expenditure for the fish, V_f = 0.3 m/s. If the water acceleration occurs from near-zero approach velocity, $a_{fw} = (V_w + V_f)(\Delta V_w/\Delta s) = (1.33 + 0.3)(1.33/0.5) = 4.34\ \text{m/s}^2$. Equation (14), for ϕ = 0, applies to this situation:

$$F_P = 0.37(1.63^{1.8}) + 1.2[69(0.261^3)/9.8] \cdot (4.34) + 69(0.261^3)(0.34) = 0.89 + 0.65 + 0.41 = 1.95\ N;$$

$$\mathrm{Pwr} = F_P(V_{fw}) = 1.95\ (1.63) = 3.17\ \mathrm{W};$$

$$E = L_e(\mathrm{Pwr})(1 + V_w/V_f) = 0.5\ (3.17)(1 + 1.33/0.3) = 8.6\ \mathrm{J};$$

for which L_e is the length of the inlet zone for the fish (0.5 m).

Because the fish moves quickly through this zone, the penalty it pays for swimming is much smaller than for slower movement through the barrel. From a very limited data base (Behlke et al. 1988), these values for F_P, Pwr, and E all appear safe. Here the inlet zone for the fish was assumed shorter than it probably would be in reality in order to illustrate the computational method. As engineers gain experience with this line of thought and the resultant numbers, the assumptions can be further refined.

Conclusions

Virtual mass and non-Archimedean buoyancy forces are shown to be of considerable importance to fish passage under certain hydraulic conditions.

The equations and procedures illustrated give bases, founded on fluid mechanics principles of fish–water interaction, for comparing one fish passage facility option with another. These procedures can be used by experienced engineers, knowledgeable about the swimming characteristics of the fish they are designing for, to design virtually any type of passage device if they also know the hydraulic characteristics of the options.

Because little data exist that can be used except for uniform flow in open channels with negligible slope, a data base of where fish actually swim in moving water masses, how fast they move with respect to the ground, what water and fish velocities and accelerations are, and the simultaneous slope of the hydraulic grade line needs to be developed. A purpose of this paper is to urge engineers knowledgeable about these parameters of fish–water interaction to publish the findings of their experiences.

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culvert slope = 0.005. Design fish is Arctic grayling 240 mm in fork length (L_f), and total length of fish (L) is assumed to be $L_f/0.92 = 261$ mm. Downstream scour pool water surface elevation will match the critical depth of flow in the culvert at the outlet for this Q. Assume velocity of fish with respect to the culvert (V_f) for short distance into culvert (0.6 m) at outlet and inlet is 0.3 m/s.

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Then,

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Experiment 2: $L = 0.76$ m, $L_c = 2.3$ m, $S_o = 0.36$, $V_{fw} = 2.77$ m/s, $V_f = 0.73$ m/s. Weight and water temperature were not given, so I assume $W = 69L^3 = 30.3$ N and a water temperature of 10°C; thus $\nu = 1.31 \times 10^{-6}$ m²/s.

Determine: F_P, Pwr, and E for each of the two experiments of Orsborn and Powers.

Calculations: Assume $a_{fw} = 0$, because no acceleration of water or fish was reported. Equation (11) describes this situation. Previous calculations have shown how to calculate D and W, so only the numerical results are shown here.

For the first experiment ($S_o = 0.27$),

$$F_P = D + W\sin(\theta) = 14.5 + 7.9 = 22.4 \text{ N};$$

$$\text{Pwr} = F_P V_{fw} = 22.4(2.68) = 60 \text{ W};$$

$$E = \text{Pwr(length of ramp)}/V_f = 60\,(2.3)/0.61 = 226 \text{ J}.$$

For the second experiment ($S_o = 0.36$), the same sequence of calculations yields

$$F_P = 15.4 + 10.5 = 25.9 \text{ N};$$

$$\text{Pwr} = 25.9\,(2.77) = 71.8 \text{ W};$$

$$E = 71.8\,(2.3)/0.73 = 226 \text{ J}.$$

In the first experiment Orsborn and Powers reported 100% of the chum salmon that attempted the chute successfully negotiated it. In the second experiment, they reported a success rate of only 23%. Because the two values for E above are identical, it is unlikely that the second set of fish were troubled by the net energy that they were capable of delivering. However, the second value for F_P is 16% greater than the first, and the second value for Pwr is 20% greater than the first. Thus, it appears most of the test fish simply could not generate enough power to deliver the propulsive force necessary to climb through the steeper chute. The reported average water velocities down the chute for each of these experiments were virtually identical, 2.07 m/s for the first and 2.04 m/s for the second, so the poor success ratio for the second experiment could not be explained by water velocity and profile drag. The above computations show the $W \sin(\theta)$ term to be 33% greater for the 0.36 slope than for the 0.27 slope, thus illustrating the importance of the fundamental fact that the buoyant force did not cancel the downslope component of the weight force.

Culvert

Hydraulically and as fish passage structures, culverts are very complicated structures, so they command great respect from design engineers. Because fish seek locations of minimum difficulty to swim, average flow velocities in culverts may not be very meaningful except as possible indices to water velocities (V_o) where the fish swim. My experience with Arctic grayling indicates that these fish swim hugging the boundary of culverts, either at the invert or close to the intersection of the water surface with the side of the culvert, whichever is the location of minimum water velocity. In short, in difficult situations that require elevated power outputs (for example, at culvert outlet and inlet), these fish seem to swim with V_f approximately equal to 0.3 m/s almost without regard to V_w so long as their anaerobic limits are not exceeded, though they may swim with much smaller values of V_f in situations of reduced power requirements (e.g., a barrel).

The horizontal angle of skew of water entering a culvert has a profound effect on the horizontal distribution of water velocity in the culvert. My measurements in one such culvert revealed that, along one boundary, water velocities 6 cm from the culvert side and 6 cm beneath the water surface (where fish swam) were only 20% of the average water velocity in the cross section. It appears, from limited observations, that if the angle of skew of water approaching the inlet is 30° to 45°, reduced wall velocity effects may be felt downstream from the culvert inlet a distance of perhaps 8 times the mean water surface width in the culvert barrel. Because this number comes from quite limited data, engineers are encouraged to observe and report their experiences with this extremely important skew effect. Clearly, culvert wall roughness greatly affects the potential for successful passage through the barrel and for success at the outlet and inlet. Multiplate culverts with 5-cm (2-in) corrugations on 15-cm (6-in) wavelengths (Manning $n = 0.035$), or other artificial or gravel–boulder roughness, generate more favorable boundary conditions than do less-roughened culverts. Waves in the culvert resulting from higher water velocities disorient small fish and frequently bounce them from slower water near the culvert wall to higher-velocity regions where they may be swept downstream.

Given: Culvert length (L_c) = 30.5 m, diameter (D) = 3.05 m, Manning $n = 0.036$, stream discharge (Q) at fish passage conditions = 2.27 m³/s, and do not accumulate in the tissues. In normal swimming the aerobic, red muscle activity is limited to long-term activity (prolonged and sustained V_{fw}). For many of the fish of interest to engineers, use of the red muscle system results in slow caudal fin movement often of large amplitude. Though this engine that the fish uses for prolonged activity delivers only small amounts of power, it can deliver a great deal of energy over a long period of time. The extent of red muscle in Arctic grayling, as an example, is exceedingly small compared with that of white muscle in the same fish. However, in the Arctic grayling, the red muscle occurs at the outer part of the body next to the skin, where it can deliver a maximum of flutter-bending moment to the caudal fin with a minimum of tension in the contracting muscle.

The white muscle engine in species that engineers are usually interested in accommodating is capable of delivering much more power than is the red muscle. Webb (1975) showed that for cold-water fish the potential power output by white muscle is approximately four times that for red muscle. White muscle functions anaerobically, however, and reactants accumulate in the muscle tissues. This manifests itself by an accumulation of lactic acid in the tissues, a product that diffuses slowly out of the muscles and, through the law of mass action and negative feedback, eventually stops further anaerobic energy production (Eckert et al. 1988). Thus, white muscle activity can only occur for a short time before a long rest is required to eliminate the lactic acid excess in the body. Burst or darting speeds can only be maintained by most species for a few seconds, and if a white muscle energy limit is reached, that muscle cannot soon be used. Negotiating a difficult culvert offers a good example of the importance of red and white muscle activity. If a fish is required to use its limit of white muscle energy to enter a culvert, it might then be able to negotiate the barrel using its red muscle system; but if the culvert inlet presents the need for burst power, the fish probably would not still have enough remaining white muscle capability to negotiate the inlet and would have to fall back downstream.

Example Calculations

Examples of fish energy and power requirements at the two extremes of upstream migration challenges are seen as fish swim through a lake or up a waterfall. Other challenges appear to lie between. Example calculations of power and energy requirements for fish swimming in a lake, a steep chute, and a culvert will be illustrated. For each situation, propulsive force (F_P), net power output (Pwr), and net energy delivered (E) will be calculated.

Lake

Given: $V_w = 0$, $V_f = 1$ m/s at time $t = 0$, $L = 0.5$ m, $b = 0.41$, $a_{fw} = 0.6$ m/s^2, $\nu = 1.55 \times 10^{-6}$ m^2/s, $\rho = 1{,}000$ kg/m^3, $W = 69(\mathrm{N/m^3}) \times L^3 = 8.6$ N, $k = 4$, gravity (g) $= 9.8$ m/s^2.

Determine: Net energy (E) delivered by the fish between times $t = 0$ and $t = 2$ s.

Calculations: Because the HGL does not slope, ∇p is directed vertically upward (equations 1, 2), and B cancels W. Only profile drag (D) and virtual mass (F_m) forces need be considered. Thus,

$$\begin{aligned} F_P &= D + F_{vm} = \text{equation (6)} + \text{equation (7)} \\ &= bk(0.072)(\rho)(\nu^{0.2})L^{1.8}V_{fw}{}^{1.8}/2 + 1.2Ma_{fw}. \end{aligned}$$

Let $C_1 = bk(0.072)(\rho)(\nu^{0.2})(L^{1.8})/2 = 1.17$. Then,

$$\begin{aligned} F_P &= C_1(V_f{}^{1.8}) + 1.2(W/\mathrm{g})(a_{fw}) \qquad (19) \\ &= C_1(1 + 0.6t)^{1.8} + 1.2(8.6/9.8)(0.6) \\ &= 4.95 + 0.63 = 5.58 \text{ newtons at } t = 2\ s; \end{aligned}$$

$$\begin{aligned} \mathrm{Pwr} &= F_P(V_{fw}) \quad [V_w = 0, \text{ so } V_{fw} = V_f = 1 + 0.6t] \\ &= 1.17(1 + 0.6t)^{2.8} + 0.63(1 + 0.6t) \\ &= 10.6 + 1.4 = 12 \text{ watts at } t = 2\ s; \end{aligned}$$

$$\begin{aligned} E &= \int_0^2 \mathrm{Pwr}\, dt \\ &= \int_0^2 [1.17(1 + 0.6t)^{2.8} + 0.63(1 + 0.6t)]dt \\ &= [1.17(1 + 0.6t)^{3.8}/3.8(0.6)] \\ &\quad + 0.63(1 + 0.6t)^2/2(0.6) \\ &= 11.8 \text{ joules net energy delivered from} \\ &\quad t = 0 \text{ to } t = 2\ s. \end{aligned}$$

Steep Channel or Chute

Reporting on live-fish experiments with chum salmon *Oncorhynchus keta* in good condition, Orsborn and Powers (1985) gave the following information for two chute studies.

Experiment 1: $L = 0.76$ m, chute length (L_c) $= 2.3$ m, slope of roughened chute (S_o) $= 0.27$, $V_{fw} = 2.68$ m/s, $V_f = 0.61$ m/s.

$$E = \int_0^t \text{Pwr}\, dt; \tag{17}$$

t is the time spent by the fish in traveling through the element. If the fish swims with constant V_f, t equals s/V_f, s being the distance traveled in the element. Thus, if Pwr is described by equation (16) and is not a function of time, equation (17) becomes

$$E = \text{Pwr}\ (V_w + V_f)(s/V_f)$$
$$= s\ (\text{Pwr})(1 + V_w/V_f). \tag{18}$$

The term V_w/V_f is the price fish pay for swimming. Tilsworth and Travis (1987) reported a 43-min travel time for passage of a single Arctic grayling *Thymallus arcticus* through a culvert of 33.5-m length, so V_f was 0.012 m/s. The water velocity where the fish swam was approximately 0.7 m/s, so $V_w/V_f = 0.7/0.012 = 56.4$. Because it swam and was unable to walk in solid contact with the culvert invert, this fish delivered at least 57.4 times as much energy as it would have if it could have walked through the culvert. (The fish's situation is similar to that of a person who is running to progress slowly upward on an escalator that is moving rapidly downward.)

Equation (18) clearly shows that the faster a fish moves through an element of a structure (fast V_f), the less energy it uses in doing so. On the other hand, it must deliver more power to move quickly through the element. My observations are that fish attempt to get through the most difficult spots in a passage structure as quickly as possible, so they seem to understand equation (18). It is also my observation that if fish cannot see the end in sight, such as in a culvert barrel, they attempt to minimize Pwr. They do so by seeking out the locations where propulsive force (F_P) is minimized, and they reduce (V_f) to some minimum consistent with forward progress, while taking their chances on being able to deliver enough energy to pass through the uncertain element.

Equations (17) and (18) contain the velocity of the fish with respect to a fixed reference (V_f). How fast fish swim in differing situations can only be learned from observations of existing situations. V_f has seldom been recorded when research results have been presented. Inspection of the above two equations clearly reveals how important it is for design engineers to have some knowledge of this parameter if they are to understand why a passage element is good or bad for fish. I suggest that engineers begin to develop catalogs of V_f for different species of fish and different sizes within a species for different difficult fish passage situations. The importance of documenting V_f for different species, sizes, and situations cannot be overemphasized.

Jones et al. (1974), in developing criteria for design of culverts on the MacKenzie Valley highway, assumed that if a fish had the capability of delivering a maximum (for the fish) instantaneous velocity with respect to the water (V_{fw}) of a given value, it actually would deliver that V_{fw} while swimming in a culvert. They then subtracted the estimated water velocity from the instantaneous value of V_{fw} and assumed the difference would be the value of V_f for fish swimming through a design culvert. I question that fish would act as they assumed, though their assumption, if adopted by the fish, would result in a minimal expenditure of energy in passing through long culverts. Fish entering a culvert do not know the culvert length until it is history to them, thus they appear to take power precautions that may or may not bring success in delivering the necessary energy to negotiate the culvert.

Some Biological Implications

The previous equations attempt to present the net propulsive force, power, and energy fish deliver if they are to pass through passage structures. It would appear that each element of the structure should be analyzed and the energy outputs should be summed to determine if the fish is capable of doing the job. However, biological constraints can confuse the efficacy of this summation. For an excellent treatise of the biological (and fluid dynamic) aspects of fish propulsion the reader is referred to Webb (1975). However, a brief, very simplified, overview of the problem is given below.

Modes of Delivery of Energy to Swimming

Fish have two muscle systems for swimming. The red muscle and white muscle systems function quite differently and are capable of delivering vastly different amounts of power and energy, depending on the species and individual fish. Webb (personal communication) described the red and white muscle systems as two different engines in one body.

Red muscle functions aerobically and depends on immediate physiological support systems. The metabolic reactants are generated in small amounts compared with the energy they release

TABLE 1.—Summary of how forces vary with fish length (L), and fluid and dynamic variables for a species of fish.

Force	Determining fish variable	Fluid or dynamic variable
Buoyant	L^3	$\nabla\rho$, γ
Profile drag	$L^{1.8}$	$V_{fw}^{1.8}$, ρ, ν
Weight	L^3	γ_f
Virtual mass	L^3	a_{fw}, ρ_f

Propulsive Force, Power, and Energy

Propulsive Force

The net propulsive force (F_P) that a fish must generate in order to pass through an element of a fish passage structure is here assumed equal to the sum of the components, in the direction of motion, of the previously discussed forces. However, in some situations the fish must also generate a propulsive component normal to the direction of motion, because of a noncancellation of weight and buoyant force components in that direction (or because of centripetal forces ignored here). That situation is probably most pronounced for fish attempting to enter the mouth of a perched culvert with subcritical flow in the barrel. For a detailed explanation of this and many of the succeeding equations see Behlke (1987) and Behlke et al. (1988).

Some examples follow of net propulsive forces a fish must deliver if it is to move ahead. If it swims upstream in an open channel, culvert barrel, or ramp where uniform, steady flow occurs, profile drag, buoyant, and weight forces impede its forward motion. Behlke (1987) showed that the fish's buoyant force acts normal to the water surface and cancels its weight component normal to the invert but does not cancel the downslope component of weight (Figure 3). Thus,

$$F_P = D + W\sin(\theta), \tag{11}$$

θ being the angle of slope of the channel, water surface, and hydraulic grade line (HGL). If a fish swims through a horizontal, baffled pipe (enclosed flow) or in an open channel inlet or outlet where it swims horizontally but the HGL slopes at angle θ, the buoyant force is greater than the fish's weight and has a downstream component that depends on the slope of the HGL. Here too, B is directed normal to the HGL; thus the propulsive force becomes

$$F_P = D + W\tan(\theta). \tag{12}$$

Where an enclosed (pipe) ladder slopes at an angle ϕ or where a fish follows a streamline at an open channel inlet or outlet that slopes at angle ϕ while the HGL slopes at angle θ,

$$F_P = D + W\{\sin(\phi) + [\cos(\phi)][\tan(\theta - \phi)]\}. \tag{13}$$

A fish attempting to enter a culvert that draws down at its outlet from subcritical flow in the barrel is attempting to move in a zone of accelerating water. If the outlet pool's elevation is sufficient to maintain culvert water depth at or above critical at the outlet lip, approximate hydrostatic conditions exist there, and the HGL slopes approximately at the slope of the water surface. Here the fish is faced also with a virtual mass force in addition to the two forces of equations (12) and (13). Thus, if the streamline (relatively straight) which the fish selects to follow slopes at angle ϕ, then

$$F_P = F_{vm} + D + W\{\sin(\phi) + [\cos(\phi)][\tan(\theta - \phi)]\}. \tag{14}$$

Inspection of this equation and the previous ones that define each of the terms in the right side of the equation (14) reveals that a good deal of information must be measured, calculated, known, or guessed before the equation can be evaluated. W requires a knowledge, usually, of the length of the smallest design fish and its weight–length characteristics. D requires the fish's length, its ratio of surface area to L^2, its velocity with respect to the moving water, and the approximate temperature of the surrounding water. F_{vm} requires the same information as that used to determine W, and it is necessary to know V_{fw} and how it changes through the acceleration zone being studied—for example the vicinity of the outlet or inlet of a culvert.

Power

The instantaneous net propulsive power (Pwr) that a fish delivers to its surroundings is

$$\text{Pwr} = F_P(V_{fw}). \tag{15}$$

Because $V_{fw} = V_w + V_f$, equation (15) can be expressed as

$$\text{Pwr} = F_P(V_w + V_f). \tag{16}$$

Energy

Net energy delivered by a fish in passing through an element of a complex passage structure is

Equations (4) and (5) assume that the fish swims directly into the oncoming current if the water is moving.

If $S = bL^2$ and equations (4) and (5) are combined, profile drag may be expressed as

$$D = bk(0.072)(\rho)(\nu^{0.2})L^{1.8}V_{fw}{}^{1.8}/2. \qquad (6)$$

Virtual Mass Force

If an object is accelerated in a fluid, if the fluid surrounding an object accelerates, or if both the object and the fluid accelerate, Newton's second law is operative. Because an object accelerating with respect to its surrounding fluid carries some of the fluid with it, an added mass is accelerated with the object. Thus,

$$F_{vm} = (M + M_a)a; \qquad (7)$$

F_{vm} is the force necessary to accelerate the object and the fluid that accelerates with it; M is the mass of the object; M_a is the added mass of the fluid accelerating with the object, and a is the acceleration of the object with respect to the surrounding fluid (Daily and Harleman 1965). The term $(M + M_a)$ is called the virtual mass of the object. In relation to fish, M_a is assumed to be approximately 0.2 M (Webb 1975). Thus, the virtual mass force which a fish feels in an acceleration situation is

$$F_{vm} = 1.2\,Ma_{fw}, \qquad (8)$$

a_{fw} being the acceleration of a fish with respect to the surrounding water. F_{vm} is in the direction of a_{fw}.

In one-dimensional motion, the acceleration term a_{fw} of equation (8) is

$$a_{fw} = V_{fw}\partial V_{fw}/\partial s + \partial V_{fw}/\partial t; \qquad (9)$$

s is distance measured along the streamline that the fish follows in swimming into the current. Usually, fish passage engineers would not be confronted with unsteady flow, so the final term of equation (9) is ignored. It then becomes necessary to evaluate the simplified form of the right side of the equation. I suggest that the equation be put in the following finite difference form:

$$a_{fw} = V_{fw}\Delta V_{fw}/\Delta s; \qquad (10)$$

ΔV_{fw} is the difference in V_{fw} between two points Δs distance apart, and V_{fw} may be approximated by the average V_{fw} over the distance Δs.

The virtual mass force usually acts against forward progress, especially at the outlets of culverts if water surface drawdown occurs there (subcritical approaching flow), at the inlet of culverts, in waterfalls where water is free-falling, and through slots and orifices where water accelerates. Also, in the leaping process fish may accelerate sharply, so during the in-water part of that process F_{vm} is an important force opposing motion.

Acceleration situations, which result in the presence of virtual mass forces, also appear to have an additional effect that hinders passage efforts of fish. Unpublished data (P. W. Webb, University of Michigan–Ann Arbor, personal communication) indicate that water or fish accelerations have some interference effects that elevate profile drag and thus may increase the value of k in equation (6) by a factor of 2 or 3. This is startling information that sounds a loud word of warning to design engineers. In most fish passage facilities the design engineer can control water accelerations in locations where fish must swim if they are to negotiate the structure. If at all possible, provisions should be made for fish to avoid zones of downstream-directed water accelerations.

Weight

Fish may change their volumes slightly, thus changing somewhat their specific weight. This process is generally slow, so it probably has little effect on most fish in passage structures. For fish of similar shape and specific weight, weight (W) is proportional to L^3. Specific weight of fish is usually assumed to be that of water.

Because buoyancy often does not completely cancel weight in fluid dynamic situations found in fish passage facilities, fish weight and buoyancy should always be considered jointly in design until it can be shown that they do cancel.

Other Forces

Yawing, centripetal, and turbulent forces and the effects of mucus on drag reduction are not considered here. I have found little information on these topics that can be converted to design principles. However, as the work of bio-fluid-dynamicists progresses, engineers can expect results that may well be incorporated in future design criteria and procedures.

The principal forces acting on fish in flowing water, and their variation with fish size, are summarized in Table 1.

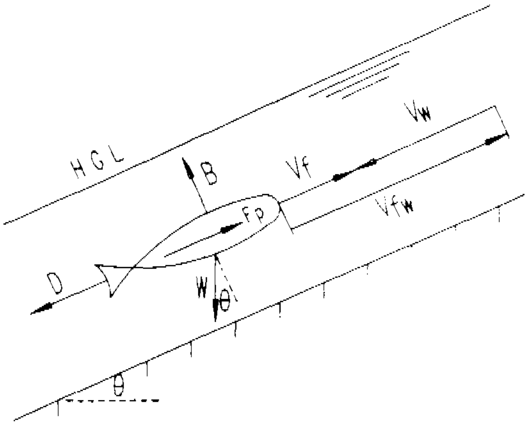

FIGURE 1.—Forces acting on fish swimming in uniform, steady flow in an open channel. B = buoyant force; D = drag; F_p = net propulsive force; HGL = hydraulic grade line; V_f = velocity of the fish; V_w = velocity of the water; V_{fw} = velocity of the fish with respect to the water; W = weight of the fish; θ = angle of the channel with respect to the horizontal.

chanics, have attempted to determine profile drag forces on fish by studying how fish appear to swim and then applying fundamental fluid mechanic concepts to evaluate quantitatively the fluid-generated drag forces on fish. These fluid-dynamicists certainly do not assert that they have the answers, but it is my observation that they, not engineers, are the leaders in this area of research. Thus, I will attempt to use their works to evaluate profile drag and to discuss later the summing of forces that a fish must overcome if it is to move through a fish passage structure.

For an outstanding treatise on swimming hydrodynamics and energetics of fish see Webb (1975). Briefly, however, biologists are presently using as a reference the drag generated by a turbulent boundary layer on a flat plate, the plate being as long as the fish and of sufficient width to have the same wetted area as that of the body of the fish. This is a severe departure from the traditional engineering use of the projected area (A) of a body in the profile drag equation,

$$D = C_d \rho A V^2/2; \qquad (3)$$

C_d is the profile drag coefficient, which depends on Reynolds number (N_R) and the shape of the body for which the drag is to be calculated; ρ is the mass density of the fluid; and V is the velocity of the fluid with respect to the body.

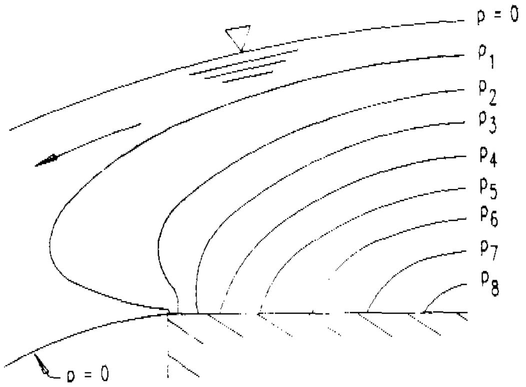

FIGURE 2.—Pressure (p) distribution in vicinity of two-dimensional over-outfall. (After Rouse 1938.)

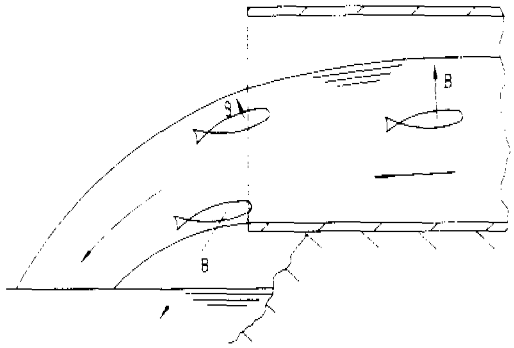

FIGURE 3.—Buoyant force (B) on a hypothetical fish at various locations in the vicinity of a free overfall at the outlet of a culvert.

Biologists (Webb 1975) have adopted as their standard (and I recommend it to design engineers) the following:

$$D = C_d \rho S V_{fw}^2/2, \qquad (4)$$

for which

$$C_d = k(0.072)N_R^{-0.2}. \qquad (5)$$

Here, S is the surface area (not projected area) of the fish, V_{fw} is the velocity of the fish with respect to the water, k is a constant that converts the reference drag coefficient to that of swimming fish, and N_R (the fish's Reynolds number) = $V_{fw}L/\nu$, ν being the dynamic viscosity of the water and L the fish's length. Biologists generally believe that the drag on a swimming fish is 3 to 5 times that of the flat-plate reference, so k varies from 3 to 5 (Webb 1975), depending apparently on the fish. Engineers might prefer 0.074 instead of 0.072 as the constant in equation (5); however, this is a minor point that is obscured by other uncertainties of the question of profile drag on swimming fish.

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Power and Energy Implications of Passage Structures for Fish

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Abstract.—Fluid mechanic equations are used to show effects of virtual mass force, non-Archimedean buoyant force, and profile drag force on fish in several fish passage structures. Example problems are worked to show computational procedures for calculating net propulsive force, net power, and net energy necessary for fish to swim in a lake, up a steep chute, and through the outlet, barrel, and inlet of a culvert.

Hydraulic Forces Affecting Swimming Fish

Buoyant Force

Fish passage engineers and others responsible for design of fish passage structures have generally assumed that fish surrounded by water are buoyed by a force equal to the weight of the volume of water displaced by the fish, and that the force is directed vertically upward. Thus, weight and buoyant forces appeared to cancel, and both were ignored. Behlke (1987) has shown that this is not the case in fish passage structures where water flows. Fundamental laws of fluid mechanics state that at any point in the fluid, the buoyant force per unit volume of fluid displaced is equal but opposite in direction to the vector gradient of the pressure, ∇p. The buoyant force (B) acting on a fish then is

$$B = (-\nabla p)(\mathrm{Vol}); \qquad (1)$$

∇p is the pressure gradient that formerly occurred in the undisturbed fluid at the instantaneous location of the swimming fish's volumetric centroid and Vol is the volume of the fish's body. In a lake, where hydrostatic pressure conditions exist, $\nabla p = \gamma$, the specific weight of water. Thus, the buoyant force would be directed vertically upward and would simply be equal to the weight of the volume of water displaced—the classical Archimedean buoyant force. However, Behlke (1987) showed that if a fish swims up through uniform, steady flow in an open channel that slopes at an angle θ with the horizontal, $-\nabla p$ is reduced from the previous value by a factor $1 - \cos(\theta)$, θ being the slope of the channel. Also, the buoyant force is directed normal to the sloping water surface (Figure 1). Thus,

$$B = \gamma(\mathrm{Vol})\cos(\theta). \qquad (2)$$

It should be noted that B is further reduced for fish swimming in aerated water because of a lesser γ.

An interesting example of how B varies in magnitude and direction is that of a free overfall (perched culvert outlet) for subcritical approaching flow. Rouse (1938) showed the pressure distribution in the vicinity of a free overfall for a rectangular channel (Figure 2). If it is assumed that a similar pressure distribution occurs in the longitudinal centerline plane of a culvert at its free overfall, resulting buoyant forces on fish attempting to swim upstream at various locations in the overfall water can be represented as in Figure 3. Clearly, buoyant forces should be considered carefully in design, and great caution must be exercised when buoyant forces acting on fish in passage structures are evaluated.

Profile Drag Force

Traditionally, engineers have been trained to evaluate drag forces on bodies of fixed shape that do not carry their propulsion systems with them. Thus, drag coefficients are determined by laboratory measurements on objects fixed in space so that force measurements are made through the fixation system. Measurements are made at constant fluid velocity. The drag, so measured, is termed profile drag. It is these drag coefficients that adorn engineering fluid mechanics textbooks. However, the development of a system for measurement of drag forces on a body that carries its propulsion system with it is an interesting, challenging, and (to the best of my knowledge) futile exercise.

Biologists have attempted to determine the profile drag on swimming fish by converting measured oxygen inputs to energy inputs (e.g., Brett 1973). However, the assumptions, calculations, and measurements necessary to get from energy input to final net propulsive force, and therefore profile drag force, have not yielded satisfactory results (Webb 1975). Thus, some biologists (and a few mathematicians), well trained in fluid me-

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