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ABSTRACT: Sonar echo integration is widely used to assess fish density when individual targets cannot be tracked or counted. We propose an echo-integration estimator of fish density, which is proportional to the ratio of the echo integral to an estimate of the average, squared echo-voltage amplitude. Our argument is based on the fact, pointed out by Ehrenberg, that amplitudes of returning echoes constitute an inhomogeneous filtered Poisson process. Our estimator, unlike the classical echo integrator, is not based on the thin-shell approximation. It uses thresholded echoes and accommodates background noises, and thus could be more appropriate in riverine environments.

INTRODUCTION

Echo integration is often used as a means to estimate fish density when individual echoes cannot be tracked or counted (MacLennan and Simmonds 1992). When a known volume of water is insonified, current is induced in the transducer by the returning echoes. Traditionally, the integrated squared voltage is scaled to the echo-integration estimate of target density using knowledge of the instrumentation, signal loss, and acoustic properties of the targets. Classical echo integration is based on a thin-shell approximation argument, hence is adapted to a marine environment, where the sonar is downward looking, and fish or plankton present homogeneous horizontal layers. We propose an alternate echo-integration estimator of fish density, proportional to the integrated squared voltage divided by the average of the squared echo amplitudes. This method estimates fish density over a thick section of the sonar beam and hence could be more appropriate to a riverine environment where sonars are side looking. We present 2 methods of calculation depending on fish density. In the low-density case, our method requires the echo integral and a sample of echo amplitudes, but it does not require information on (1) beam pattern factor or the equivalent beam angle, (2) sound attenuation parameters by the medium or from propagation, or (3) the distribution or mean value of the back-scattering cross-section of the insonified fish. In the high-density case our method depends on these parameters but is applicable when echoes overlap. Background noises that usually corrupt the echoes are taken into account in our method.

Our key result is Eq. (16), which describes an approximate relationship between the mathematical expectation of the echo integral and target density, the expected value of the squared echo amplitudes, and known constants.

We do not need to know the spatial location of the targets, only that the targets are within a specified volume of water. Hence this estimation procedure applies to all types of sonar as long as the precise beam geometry is known.

This work can be considered a continuation of the work of Moose (1971), Moose and Ehrenberg (1971), and Ehrenberg (1973), who essentially obtained Eq. (12), but did not make full use of echo amplitude information in their echo integration estimators. Their methods rely on calibrations to scale the echo integrator.

METHODS

Assumptions

The *insonified volume* is the volume of water in which sound from one ping propagates. This volume is comprised of a main lobe and side lobes. The *gated volume* is a portion of the insonified volume bounded between the spheres centered on the transducer, with

Authors: PHAM X. QUANG did this work while he was with the Department of Mathematical Sciences, University of Alaska, Fairbanks. His current address is 1006 Cinnabar Way, Vacaville, CA 95687. E-mail: quang@stat.berkeley.edu. HAROLD J. GEIGER works for the Alaska Department of Fish and Game, Commercial Fisheries Division, P.O. Box 240020, Douglas, AK 99824. E-mail: hal_geiger@fishgame.state.ak.us
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radii R_1 and R_2. The gated volume has approximately the shape of the frustum of a cone with a plane circular or elliptical base, depending on the type of sonar. We chose R_1 and R_2 such that the gated volume was carved out of the main lobe only and was located in the far-field (the Fraunhofer zone) of the sonar. The gate times in the sonar integrator are set to t_1 and t_2 such that $t_1 = 2R_1/c$ and $t_2 = 2R_2/c$, where c is the sound speed. Our primary purpose was to obtain a statistical estimate of the density of the fish within the gated volume that returns echoes from a single ping. We then improved that estimate by using echoes returned from a short string of pings. We assumed:

1) The reverberations are only from the fish, not from debris, logs, motor boat wakes, or river bottom.
2) External noises (air bubbles, water turbulence) and noises from the sonar circuitry are modeled as a Wiener process, $e(t)$, that is independent of the number of fish in the beam and their position.
3) Fish in the gated volume are not so numerous as to occult each other.
4) Doppler shift due to swift movements of fish or current is negligible.
5) Fish are randomly distributed within the gated space; their number at any given moment is Poisson distributed.
6) The milieu is isotropic; sound waves propagate spherically.
7) Emitted pings have a rectangular envelope; returning echoes have the same duration as the ping and also have a rectangular envelope, although with different amplitudes.
8) The electrical response of the system to sound pressure is linear; voltage outputs at the transducer are proportional to echo amplitudes.
9) No fish has an echo with amplitude below a threshold value.

Regarding the sonar, we assumed:

10) The precise geometry of the beam is known.
11) The duration t and frequency f of a ping are known.
12) The value of the speed of sound in water c is known.
13) In response to a ping, each fish emits one echo, and the sonar receives all echoes from a given ping.
14) Ping frequency is low enough to record all echoes from a ping before the next ping is emitted.

Note that we allow echo interferences at the level of the transducer and do not assume that the echoes have random phases, as most authors do.

Our calculations assume negligible fish movement during the lifetime of a ping. We could argue that such is the case in a river environment because a ping and its train of echoes occur over a period of only about 0.01 second.

Assumption (5) is central to our calculations. Fish, especially migrating salmon, are not randomly distributed in the volume of the river. But the main beam of a sonar usually is a relatively "thin pencil," hence randomness of fish location within the beam holds, at least approximately.

Assumption (9) allows us to filter out low amplitude echoes we believe come only from water turbulence or air bubbles, but not from the fish.

Assumption (13) fails if an echo from one fish reflects on another. Our fish density estimate would be biased high if multiple paths exist.

Echo Integral

In this section we present the sonar equation, the echo integral and its mean value. Following Moose and Ehrenberg (1971), consider a ping emerging from the transducer at time 0 and one fish located at coordinates (R, q, f). The distance from the transducer to the fish is R, q is the off acoustic-axis angle or colatitude, and f is the polar angle of projection on the xy plane or longitude. Here x is directed upstream, y is directed vertically up, z is the acoustic axis and perpendicular to the stream. We write the ping pressure as a function of time

$$p_0(t) = P_0 \sin(wt)\mathbf{I}(0 \le t \le t) \tag{1}$$

where $\mathbf{I}$ is the indicator function

$$\mathbf{I}\left(0 \le t \le t\right) = \begin{cases} 1 & \text{if } 0 \le t \le t \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

Implicit in this equation is the assumption (following Moose and Ehrenberg 1971 and Medwin and Clay 1997) that all pings are rectangular and have the same duration t, frequency w, and amplitude P_0.

An echo from a fish propagates back to the transducer. Its pressure at the transducer follows equation 9.2.4 of Medwin and Clay (1997) that we rewrite with slightly different notations:

$$p(t) = P_0 b^2(q, f)\frac{10^{-aR/10}}{R^2}\sqrt{s_{bs}} \sin\left[w\left(t - \frac{2R}{c}\right)\right]\mathbf{I}\left(0 \le t - \frac{2R}{c} \le t\right). \tag{3}$$

Here s_{bs} is the back-scattering cross-section of the fish, a is the plane-wave attenuation rate, and $b(q, f)$ is the beam pattern factor.

If $h=2R/c$, then h is the roundtrip transducer-fish-transducer travel time of the sound. Eq. (3) becomes

$$p(t) = P_0 b^2 (q, f) \frac{10^{-aR/10}}{R^2} \sqrt{s_{bs}} \sin\left[w\left(t-h\right)\right] \mathbf{I}\left(0 \le t-h \le t\right). \tag{4}$$

The voltage output induced by the echo is proportional to pressure:

$$v(t) = CP_0 b^2 (q, f) \frac{10^{-aR/10}}{R^2} \sqrt{s_{bs}} \sin\left[w\left(t-h\right)\right] \mathbf{I}\left(0 \le t-h \le t\right). \tag{5}$$

The value of C depends on the physical characteristics of the sonar and may include a fixed voltage gain. We refer to Eq. (5) as the sonar equation.

Many authors, including Ehrenberg, Medwin and Clay, MacLennan and Simmonds, would rewrite their Eq. (5) as

$$v(t) = CP_0 b^2 (q, f) \frac{10^{-aR/10}}{R^2} \sqrt{s_{bs}} \sin\left(wt - x\right) \mathbf{I}\left(0 \le t-h \le t\right) \tag{6}$$

and assume that the *phases* x corresponding to all insonified fish are mutually independent and have a uniform distribution over $[-p, p]$. This directly leads to the verification of the *linearity hypothesis in echo integration* (e.g., see Foote 1983; MacLennan 1990). Such an assumption is not required in our work.

For ease of notation, we set:

$$X = CP_0 b^2 (q, f) \frac{10^{-aR/10}}{R^2} \sqrt{s_{bs}}, \text{ and} \tag{7}$$

$$g(t, h) = \sin\left[w\left(t-h\right)\right] \mathbf{I}\left(0 \le t-h \le t\right). \tag{8}$$

The sonar equation now can be compactly written as

$$v(t) = Xg(t, h). \tag{9}$$

Let $N(t)$ be the number of fish being insonified over the time interval $[0, t]$. These are fish located at the intersection of the insonified volume and the sphere centered on the transducer with a radius $^1/_2\, ct$. If we assume fish density r is the same at all locations in the beam, then $N(t)$ is an inhomogeneous Poisson process with intensity $l(t)$. For t in the range $[t_1, t_2]$ we have

$$l(t) = \frac{1}{8} r t^2 c^3 p \sin q_1 \sin q_2, \tag{10}$$

where $2q_1$ and $2q_2$ are the left-right and up-down angles of the main lobe. The volume of the intersection between the main lobe and the sphere with radius $^1/_2\, ct$ is approximately the frustum of a cone with an elliptical base and equals $^1/_{24}(ct^3)p \sin q_1 \sin q_2$. The mean number of fish in that volume is

$$\Lambda(t) = \frac{1}{24} r (ct)^3 p \sin q_1 \sin q_2.$$

$N(t)$ has a Poisson distribution with mean $\Lambda(t)$, hence $l(t) = d\Lambda(t)/dt$ has the required form.

The total voltage output at time t produced by all echoes from the same ping and background noise is:

$$V(t) = \sum_{m=1}^{N(t)} CP_0 b^2 (q_m, f_m) \frac{10^{-aR_m/10}}{R_m^2} \sqrt{s_{bs,m}} \sin\left[w\left(t-h_m\right)\right] \mathbf{I}\left(0 \le t-h_m \le t\right) + e(t), \tag{11}$$

where $e(t)$ has an unknown variance h. Here R_m, q_m, and f_m denote the spherical coordinates of the mth insonified fish; $s_{bs,m}$ is its back-scattering cross-section; and $h_m = 2R_m/c$ is the arrival time of its echo. In a more compact notation:

$$V(t) = \sum_{m=1}^{N(t)} v_m(t) = \sum_{m=1}^{N(t)} X_m g(t, h_m) + e(t) \tag{12}$$

with

$$X_m = CP_0 b^2 (q_m, f_m) \frac{10^{-aR_m/10}}{R_m^2} \sqrt{s_{bs,m}}. \tag{13}$$

In Eq. (12) we recognize X_m as the amplitude of the envelope voltage of the echo from the mth fish, and that X_m is a *recorded and available datum*, even though its expression in Eq. (13) contains quantities whose numerical values may be difficult to obtain. Moreover, $\sum_{m=1}^{N(t)} X_m g(t, h_m)$ is a filtered inhomogeneous Poisson process.

The echo integrator within the sonar circuitry provides the quantity

$$I(t_1, t_2) = \int_{t_1}^{t_2} V^2(t)\, dt. \tag{14}$$

Here $t_1=2R_1/c$ and $t_2=2R_2/c$, where R_1 and R_2 delimit the gated volume.

Because t_1 and t_2 are fixed throughout, we write I for $I(t_1, t_2)$. We recognize, from a statistical standpoint, that the quantities R_m, q_m, f_m, $s_{bs,m}$, X_m, and $N(t)$ are random. Hence I is random.

Mixed moments of $V(t)$, such as $E[V(t)]$, or $E[V(t_1)V(t_2)]$, that we need for statistical inference can be calculated when the joint characteristic functions of $V(t_1)$ and $V(t_2)$ are known. Snyder and Miller (1991) establish such joint characteristic functions under the causality condition of the filtered inhomogeneous Poisson process. The causality condition, which is satisfied in our case, is $X_m g(t,h_m) = 0$ for $t < h_m$. That is, the response cannot occur before the occurrence of the impulse. Expressions of mixed moments (except $E[V^2(t_1)V^2(t_2)]$, which is necessary for calculating the variance of the echo integral), can be found in exercise 5C, page 156 of Parzen (1962). Extensive moment calculations can also be found in Ehrenberg's 1971 thesis (Ehrenberg 1971).

The second moment of X plays a key role in what follows. We use the traditional notations

$$m_1 = E\left(X\right) \text{ and } m_2 = E\left(X^2\right). \qquad (15)$$

Now using the results from these authors, we obtain:

$$\begin{aligned} E\left[V\left(t\right)\right] &= m_1 \int_0^t l\left(h\right) g\left(t,h\right) dh \\ &= m_1 \int_{t-t}^{t} l\left(h\right) \sin\left[w\left(t-h\right)\right] dh \\ &= r A_1\left(t\right) m_1, \end{aligned}$$

$$\begin{aligned} \operatorname{var}\left[V\left(t\right)\right] &= m_2 \int_0^t l\left(h\right) g^2\left(t,h\right) dh + h \\ &= m_2 \int_{t-t}^{t} l\left(h\right) \sin^2\left[w\left(t-h\right)\right] dh + h \\ &= r A_2 m_2 + h, \end{aligned}$$

and

$$\begin{aligned} E\left(I\right) &= \int_{t_1}^{t_2} E\left[V^2\left(t\right)\right] dt = \int_{t_1}^{t_2} \left[\operatorname{var} V\left(t\right) + E^2 V\left(t\right)\right] dt \\ &= r m_2 \int_{t_1}^{t_2} A_2\left(t\right) dt + \left(t_2 - t_1\right) h + r^2 m_1 \int_{t_1}^{t_2} A_1^2\left(t\right) dt \\ &= r m_2 B\left(t_1,t_2\right) + \left(t_2 - t_1\right) h + r^2 m_1^2 B_1\left(t_1,t_2\right). \end{aligned}$$

Here

$$A_r\left(t\right) = \frac{1}{8} c^3 p \sin q_1 \sin q_2$$

$$\int_{t-t}^{t} h^2 \sin^r\left[w\left(t-h\right)\right] dh \text{ for } r = 1, 2$$

and

$$B\left(t_1,t_2\right) = \frac{1}{48} p\, r c^3 \left(t_2^3 - t_1^3\right) \sin q_1 \sin q_2$$

and

$$B_1\left(t_1,t_2\right) = \int_{t_1}^{t_2} A_1^2\left(t\right) dt.$$

Under usual operating conditions, the quantity $B_1(t_1, t_2)$ is negligible compared to $B(t_1, t_2)$. In particular, this is the case when t is very small compared to t_1. For example, in the Deep Creek 1996 experiment (Iverson 1996), a Model 240 split beam system from Hydroacoustic Technology, Inc., was used. One of the transducers produced a 2.8°×10° beam with a range of about 10 m. The ping had frequency f=200 kHz and adjustable duration between 0.100 msec and 1 msec. The ping rate was 30 pings per second. If t = 0.5 msec and sound speed c=1500 m/sec, then $w = 2pf$= 1,256,637 radians/sec. If the gated volume is the part of the beam between R_1= 5 m and R_2= 8 m, we obtain $B_1(t_1, t_2) = 2.273\times10^{-12}$ and $B(t_1, t_2) = 9.570\times10^{-5}$. Here t_1= 2×5/1500×5 = 6.7 msec and t_2= 10.7 msec are much larger than t = 0.5 msec.

Neglecting the term $B_1(t_1, t_2)$ in the expression of $E\,(I)$ is equivalent to neglecting the "cross-product term" in traditional echo integration, and is called the *linearity principle.*

Thus, we can write an excellent approximation:

$$\begin{aligned} E\left(I\right) &= \frac{1}{48} p\, c^3 \left(t_2^3 - t_1^3\right) \sin q_1 \sin q_2\, r\, t\, m_2 \\ &+ \left(t_2 - t_1\right) h. \end{aligned} \qquad (16)$$

Let $K\left(t_1,t_2\right) = \frac{1}{24} p\, c^3 \left(t_2^3 - t_1^3\right) \sin q_1 \sin q_2$ be the volume of the gated space. Then Eq. (16) can also be written as:

$$E\left(I\right) = \frac{1}{2} K\left(t_1, t_2\right) r\, t\, m_2 + \left(t_2 - t_1\right) h. \qquad (17)$$

Eq. (17) has a very simple form. We note in particular that the random quantities x and $N(t)$ have been nicely dealt with. We will use Eq. (17) to estimate r in the next section.

Moose and Ehrenberg (1971) obtain a result similar to Eq. (17), except that they use a time varied gain (TVG)-corrected version of Eq. (11):

$$V\left(t\right) = \sum_{m=1}^{N(t)} F_m \sin\left(wt - x_m\right) \mathbf{I}\left(0 \le t - h_m \le t\right), \qquad (18)$$

where F_m is the TVG-corrected voltage output, and they assume that all phases x_m are stochastically independent. They noted that, in their version of Eq. (17), $E(X^2)=tE(s_{bs})$, and that this could be a basis for the estimation of r if the mean back-scattering cross-section $E(s_{bs})$ of the fish within the beam is known.

RESULTS

Estimation of Fish Density

First we estimate the Wiener process variance, h, by running the sonar when fish are not present, while making sure that all climatic and tidal conditions are the same as when fish are present. Then h can be estimated as the sample variance, $\hat{h}$, of the recorded values of $V(t)$.

Now we propose 2 methods to estimate the fish density r, the first of which does not require knowledge of the mean back-scattering cross-section of the fish in the beam, the equivalent beam pattern factor, and the attenuation coefficient, but is applicable only under low fish density. The second method depends on these parameters but is applicable under any density level.

Estimation at Low Density

Assume low density so that only a few fish are in the sonar beam at any given time so their echoes have little chance of overlapping, and hence their X-values can be measured unambiguously. In Eq. (17) all quantities except r are known, and I and X are observable repeatedly. Hence Eq. (17) can serve as the basis for estimating r as follows.

Suppose that n pings were emitted over a period of time and that fish density r remains constant during that period. The assumption of constant fish density may seem reasonable if the time period was short enough. Assume also that each echo can be unambiguously associated with the ping that produced it. This is the case if all echoes from a ping are recorded before the next ping is emitted. In the Deep Creek experiment, 1800 pings were emitted in a minute. Note that the time interval between 2 consecutive pings, $^1/_{30}$ = 0.033 seconds, is much larger than the roundtrip transducer-fish-transducer of the sound which is at most 2 × 10/1500 = 0.0133 seconds. Hence the transducer will have received all echoes from a ping before the next ping is emitted. Corresponding to the ith ping, we have the echo integral I_i and t_i voltage amplitudes of the echoes $X_{i1}, ..., X_{ir_i}$. The data that are available are shown in Table 1.

Table 1. Echo interval and voltage amplitudes.

Ping No.	Echo Integral	Voltage Amplitude
1	I_1	$X_{11}, ..., X_{1r_1}$
2	I_2	$X_{21}, ..., X_{2r_2}$
...	...	...
n	I_n	$X_{n1}, ..., X_{nr_n}$

If

$$\bar{I} = \frac{1}{n}\sum_{i=1}^{n} I_i, \quad U_i = \frac{1}{r_i}\sum_{j=1}^{r_i} X_{ij}^2, \text{ and}$$

$$\hat{m}_2 = \bar{U} = \frac{1}{n}\sum_{i=1}^{n} U_i, \text{ and} \tag{19}$$

$$s_I^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(I_i - \bar{I}\right)^2, \text{ and}$$
$$s_U^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(U_i - \bar{U}\right)^2, \tag{20}$$

then we propose estimating r by:

$$\hat{r} = \frac{2\bar{I} - 2\left(t_2 - t_1\right)\hat{h}}{t\,K\left(t_1, t_2\right)\hat{m}_2}. \tag{21}$$

Using the delta method, and noticing that $\mathrm{cov}\left(\bar{I}, \bar{U}\right) = \frac{1}{n}\ \mathrm{cov}\left(I_1, U_1\right)$ is negligible when n is large:

$$v\hat{a}r\left(\hat{r}\right) = \frac{4}{nt^2K^2\left(t_1, t_2\right)}\left(\frac{s_I^2}{\hat{m}_2^{\,2}} + \frac{\left[\bar{I} - \left(t_2 - t_1\right)\hat{h}\right]^2 s_U^2}{\hat{m}_2^4}\right). \tag{22}$$

Regarding the quantities X_{ij}^2s, note that $\hat{m}_2$ is only their mean. If some echoes overlap, the X_{ij} may be difficult to read. We can safely skip these. Because hydroacoustic measurement produces so much data, even with drastic selecting and sampling the law of large numbers can ensure estimates of b with virtually no sampling error for extensive surveys. In an actual hydroacoustic assessment, nonsampling errors provide a much greater source of uncertainty in fish density estimates.

Estimation at High Density

When there are many fish inside the sonar beam, some echoes will overlap, hence it may be difficult to read the amplitudes X of the individual echoes. We can still use Eq. (17) to estimate r, but m_2 must now be estimated otherwise than by sampling the X_{ij}s. We obtain from Eq. (7):

$$m_2 = C^2 P_0^2 E\left[b^4(q,f)\frac{10^{-aR/5}}{R^4}\right]E(s_{bs}) \qquad (23)$$

because s_{bs} is independent of the triple (R, q, f). Now the quantity $E(s_{bs})$ can be estimated by the mean backscattering section $\overline{s_{bs}}$ that is given by the sonar system if target strengths (TS) expressed in decibels are available, then $\overline{s_{bs}}$ is the average of the quantities $10^{TS/10}$, and the quantity $E[b^4(q, f)10^{-\alpha R/5}/R^4]$, which is a mathematical expectation, can be estimated by an average of the observed values of $b^4(q, f)10^{-\alpha R/5}/R^4$ (calculated using spherical coordinates of the fish in the beam and the value of the attenuation parameter a). Then an estimate $\tilde{m}_2$ of m_2 is the product of these estimates. The fish density r can now be estimated by:

$$\tilde{r} = \frac{2\bar{I} - 2(t_2 - t_1)\hat{h}}{t\,K(t_1, t_2)\,\tilde{m}_2}. \qquad (24)$$

Note that this method of estimation is applicable whether fish density is high or low. It requires knowledge of a and the analytic expression of $b\,(q, f)$.

DISCUSSION

By reexamining the basic principles supporting echo integration we hope to make the theory more accessible and to find ways to use echo integration ideas in what we have come to call hybrid algorithms of sonar fish density. Many may find it surprising that echo integration is possible without knowledge of mean target strength, equivalent beam angle, or an attenuation coefficient. This approach to echo integration has been difficult for us to explain. Many readers probably feel that echo integration works as well as possible in its traditional form (MacLennan and Simmonds 1992), even though the ideas behind echo integration remain a mystery to many. Traditionally, echo integration is presented as a blend of exotic alphabets, with each symbol representing the result of "a major topic in itself," in the words of MacLennan and Simmonds. Eq. (21) is interesting for the few assumptions that go into it. Naturally, practical echo integration will require careful understanding of the actual nature of amplitude envelope voltage X_{ij} and a more complex treatment of these values than we have supplied here. For example, at very high density echoes will overlap, and not all values of X_{ij} will be available or suitable for use to estimate m_2. In actual practice, sampling and filtering would lead to better estimates of m_2 and provide improvements to the straightforward estimates of fish density r that we have provided.

Both Eq. (21) and Eq. (24) offer a straightforward means to estimate r. They differ in how they acquire their estimate of m_2. A preference for Eq. (24) over Eq. (21) depends on the ability to accurately know systems parameters. It remains to be seen which is preferred, but Eq. (24) allows the algorithm a starting point, even before acquiring any data.

Target-tracking and echo-counting estimates of density will begin to fail and show a downward bias at very high density (one possible reason is that the system is incapable of distinguishing echoes that overlap). In riverine applications, we will occasionally face the problem of estimating fish densities at these high levels. Our working hypothesis has been that we can find an imprecise echo integration estimator of fish density that will maintain a linear trend at densities where echo counting or target tracking begin to show unacceptable bias. If so, one idea for a hybrid algorithm would be to simply take a weighted average of a target-tracking estimate of fish density and an echo-integration estimate, with the weights near one and zero until density becomes very high. Another idea would be to develop an empirical functional relationship between a target-tracking estimate of fish density and an echo-integration estimate at low and moderate densities, and then use the relationship to bias correct at high densities. Echo integration will undoubtedly be imprecise in a riverine application, but it remains to be seen if we can overcome the practical problems with its use.

LITERATURE CITED

Ehrenberg, J. E. 1971. Estimation of the intensity of a filtered Poisson process and its application to acoustic assessment of marine organisms. Doctoral dissertation, Department of Electrical Engineering, University of Washington, Seattle.

Ehrenberg, J. E. 1973. Estimation of the intensity of a filtered Poisson process and its application to acoustic assessment of marine organisms. Washington Sea Grant Publication WSG 73-2, University of Washington, Seattle.

Foote, K.G. 1983. Linearity of fisheries acoustics, with addition theorems. Journal of the Acoustical Society of America 73:1932–1940.

Iverson, T. K. 1996. Feasibility of using split-beam hydroacoustics to monitor adult Chinook salmon escapement in Deep Creek, Alaska. Hydroacoustic Technology, Inc., Seattle, Washington.

MacLennan, D. N. 1990. Acoustical measurement of fish abundance. Journal of the Acoustical Society of America 87:1–15.

MacLennan, D., and J. Simmonds. 1992. Fisheries acoustics. Chapman and Hall, New York.

Medwin, H., and C. S. Clay. 1997. Fundamentals of acoustical oceanography. Academic Press, New York.

Moose, P. H. 1971. A simplified analysis of the statistical characteristics of the fish echo integrator. Washington Sea Grant Publication WSG 71-2, University of Washington, Seattle.

Moose, P. H., and J. E. Ehrenberg. 1971. An expression for the variance of abundance estimates using a fish echo integrator. Journal of the Fisheries Research Board of Canada 28:1293–1301.

Parzen, E. 1962. Stochastic processes. Holden-Day, San Francisco.

Snyder, D.L., and M. I. Miller. 1991. Random point processes in time and space. Springer Verlag, New York.

Appendix A. List of symobls used in the text and equations.

a	sound attenuation coefficient
h	variance of e (t)
$\hat{h}$	estimate of h
$e(t)$	Wiener process representing the noise component of voltage output
$l(t)$	Poisson intensity
$L(t)$	Poisson mean
μ_1, μ_2	first and second moment of X, $\mu_1 = E(X)$, $\mu_2 = E(X^2)$
$\hat{m}_1, \tilde{m}_1$	estimates of μ_1
r	fish density
$\hat{r}, \tilde{r}$	estimates of r
q_1, q_2	left-right and up-down angles of the main beam
t	length of ping and echoes
q, f, R	spherical coordinates of a fish
s_{bs}	back-scattering cross-section of a fish
x	phase difference
w	sound frequency
$b(q, f)$	beam pattern factor
A	place holder for complicated expression
$B(t_1, t_2)$	the expression $B(t_1, t_2) = \frac{1}{48} p t c^3 (t_2^3, t_1^3) \sin q_1 \sin q_2$
c	speed of sound in water
C	pressure-to-voltage output of the *mth* echo
E	mathmetical expectation
F_m	TVG-corrected voltage output of the *mth* echo
$g(t, h)$	the expression $\sin[w(t-h)]\mathbf{I}(0 \le t-h \le t)$
h	transducer-fish-transducer travel time of sound, $h = 2R/c$
$\mathbf{I}$	indicator function
I	the echo integral over gated space
$K(t_1, t_2)$	volume of the gated space
m	index referring to the *m*th fish
$N(t)$	number of fish insonified at time t
$p_0(t)$	ping pressure at face of transducer
$p(t)$	ping pressure at fish location
P_0	ping amplitude
s^2	sample variance
TS	target strength
U	mean squared voltage amplitude
$\bar{U}$	sample mean of U
$v(t)$	voltage output of an echo
$V(t)$	total voltage output of all threshold echoes for one ping
X	voltage amplitude of an echo

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