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A COMPOUND MULTIVARIATE BINOMIAL-HYPERGEOMETRIC DISTRIBUTION DESCRIBING CODED MICROWIRE TAG RECOVERY FROM COMMERCIAL SALMON CATCHES IN SOUTHEASTERN ALASKA

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ABSTRACT

The increasing production of Southeastern Alaska hatchery facilities and the difficulties inherent in management of mixed hatchery and wild stock fisheries require that managers have accurate and timely estimates of the number of hatchery fish in a harvest. Coded wire tagging and sampling programs are used for this purpose. The development of statistical methodology appropriate to these programs is necessary not only to derive estimates of the contribution of hatchery releases to the fisheries, but also to optimize the allocation of limited resources to different programs and sections of these programs, and to anticipate the effects of modifications of tagging and sampling goals on the precision of the estimates. Such statistical methodology is developed in the present study. A compound binomial-hypergeometric distribution is proposed as an appropriate model to describe the Southeastern Alaska tagging and sampling program. The probability of recovering and decoding a given number of coded wire tags of a unique tag code is dependent on the proportion of a release tagged, the proportion of catch sampled, the number of fish heads lost prior to arrival at the tag lab, the number of tags lost prior to decoding, and the number of fish of the given release in the catch. The multivariate forms of the hypergeometric distributions in the model are used to derive an equation for the covariance between recoveries of two different tag codes in the same sampling stratum. Comparison of estimates calculated using this model with empirical estimates of replicate tagged releases from Whitman Lake hatchery suggest that the model accurately estimates the variance. In general, sampling and tagging proportions should be similar in magnitude in order to minimize the variance of the estimates. Increasing sampling effort will only partially compensate for a small proportion of tagged fish in a release. For a given funding level and unit costs of a project, methods to approximate the number of fish to tag and fish to sample in order to maximize precision are derived. The level of funding needed to realize a given level of precision can also be calculated. Stratification of a sampling program will also contribute to the variance of contribution estimates. Optimum allocation of a given level of tagging or sampling effort across releases or sampling strata can be found by a series of ratios. Although the statistical methodology was developed to describe specifically the Southeastern Alaska coded wire tagging and sampling program, results can, with some modifications, be extrapolated to other west coast programs.

Key Words: Coded Wire Tag, Sampling Levels, Compound Distribution, Binomial, Hypergeometric, Variance Covariance.

INTRODUCTION

In 1982, approximately 1,163 chinook salmon (*Oncorhynchus tshawytscha*) of Alaska hatchery origin were harvested in Southeastern Alaska commercial fisheries (Clark et al, 1985). In 1985, Alaskan hatcheries contributed an estimated 10,656 chinook salmon to the commercial catches (unpub. data; preliminary results presented to Joint U.S.-Canada Chinook Technical Committee). By 1995, increased production of both state and private non-profit facilities is projected to result in over 200,000 chinook salmon being available for commercial harvest (ADF&G, 1984), or a 64% increase in the 1970 to 1982 average annual catch of 312,357 chinook salmon. Similar increases in both chum (*O. keta*) and coho (*O. kisutch*) salmon hatchery production will increase the number of hatchery fish available for commercial harvest to levels comparable with or exceeding those of natural stocks (Fig. 1). The 674,300 hatchery coho salmon forcasted to return in 1994 will result in a 53% increase above the average annual commercial catch of coho salmon from 1970 to 1984. The 4,482,400 chum salmon which are projected to return in 1992 are 2.8 times the average 1970 to 1984 commercial catch.

Enhancement activities are used to mitigate fish losses from foreign interceptions, environmental disruptions, and international treaty limitations; to supplement the harvest of natural stocks in depressed fisheries; and to increase the catches of and create new opportunities for existing commercial, sport, and subsistence fisheries (Hansen, 1985). However, when enhancement activities artificially produce salmon at levels comparable with or exceeding wild stock production, indiscriminate harvest of mixed wild and hatchery fish will almost certainly result in overharvest of the natural stocks. Ricker (1958) and Paulik et al (1967) discussed the consequences of harvesting two or more unequally productive populations at a common rate of removal. Because hatcheries can produce fish at rates many times those of natural stocks and survival rates of hatchery stocks can be several times those of natural stocks, the same concerns have been expressed by Larkin (1979; 1981); Hankin (1982); Wright (1981), and others regarding the harvest of hatchery and wild stocks in a common fishery. Continued harvest of fish of hatchery and natural stock origin at rates which optimize the levels of exploitation of hatchery returns but exceed the levels that wild stocks can withstand will lead to declining abundance and possible extinction of natural stocks. The decline in Oregon coastal coho salmon

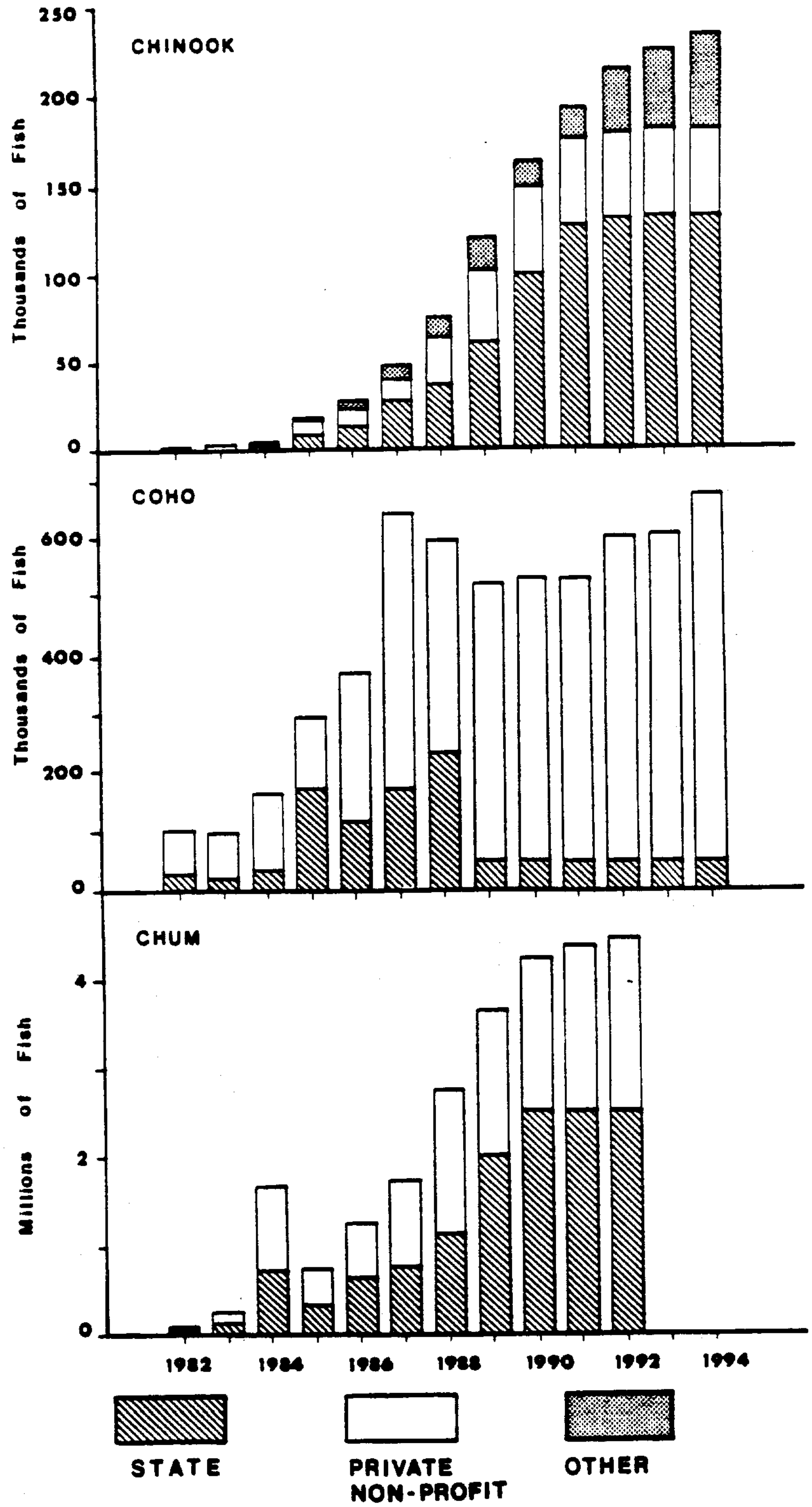


Figure 1. Estimated (1982 and 1983 chinook salmon) and projected (other years and species) returns of hatchery produced chinook, coho, and chum salmon from 1982 to 1994. 'Other' refers to Federal and Annette Island production. Projected returns are from ADF&G (1984), and estimates are from Clark et al (1985) and Marshall and Clark (1986).

stocks may be in part due to increased releases from public hatcheries and the corresponding increase in harvest rates by commercial and sport fisheries (Scarnecchia and Wagner, 1980).

In a mixed hatchery-wild stock fishery, a primary goal of fisheries management is the adequate escapement of natural stocks while allowing for a maximal harvest of hatchery fish. To achieve this, management must accurately discriminate between hatchery and wild stocks in mixed stock harvests. Scale pattern analysis has been shown to be of some value in differentiating between adult wild and hatchery coho salmon (Scarnecchia and Wagner, 1980). However, coded wire tagging of hatchery releases is the most widely used means of identifying hatchery fish. Coded Wire Tags (CWTs) are 1 mm long microscopic wires which are binary coded (newer CWTs may be half length and contain either color or rare earth codes) and implanted into the nose cartilage of juvenile salmon. The adipose fin of the tagged salmon is removed to identify the adult fish which possess CWTs. By agreement, the adipose clip is reserved to indicate the presence of a CWT in a salmon. In addition to providing estimates of the number of hatchery fish in a time-area-fishery sampling stratum, analysis of CWT data also help provide information concerning the survival, growth, age at maturity, migratory timing, and direction of travel of hatchery releases.

Data on CWT recoveries in Canadian and State of Washington fisheries have been compiled and documented in a series of reports (Bailey et al, 1984; Bailey et al, 1983; Simpson et al, 1981a and 1981b; Heizer, 1978; Cook et al, 1979; Cook et al, 1978; and O'Connor and Packer, 1983). Each report details tag recovery effort and estimates the total number of tags recovered of each tag code by area, fishery, and time strata. Such information is useful to hatchery personnel for comparison of the biological attributes of different releases. However, management of wild and hatchery mixed stock fisheries requires estimates of the total number of hatchery fish in a time-area-gear stratum (total number of tags expanded to account for the untagged proportion of a hatchery release) and the associated accuracy of the estimates. <u>An estimate of the total number of CWTs of a given tag code or the total number of fish of a given release represented by that tag code in fishery catches without a measure of the accuracy or precision of the estimate is of little value and may be misleading in strata of small sampling effort or for releases with a small proportion of the release tagged!</u>

Several methods have been proposed to calculate the variance of CWT returns. Neeley (1982) resolved the estimation procedure into its component parts and presented rules governing the estimation of the total variance based on the functional relation of each procedure component. The variance of the number of tags recovered was calculated by summing the squared deviation of the number of tags recovered in each landing from the mean number of tags recovered over all landings sampled, and dividing by the number of landings sampled minus 1. Multiplication by a finite population correction factor was suggested. Variation in the expansion factors increases the variance of the contribution estimates according to formulas derived by Cochran (1953) and Goodman (1960) and presented by Neeley (1982). The same approach was used by Webb (1985) except the negative binomial distribution was recommended as the preferred means of obtaining the variance of the number of tags recovered. Hankin (1982) developed estimators based on the ratios of adclipped CWTed fish and non-adclipped marked fish returning to the same river drainage. Variances were calculated by summing the component variances of assumed independent sampling events.

No study has developed a strong statistical foundation for the methods used to estimate the contribution and associated variance of a tagged release to a fishery catch stratum. Equations which estimate the contribution have been created on the basis that they are intuitively correct. Potential problems of bias in the estimation proceedures are generally ignored (however, see de Libero (1986) for an excellent discussion on the types of errors associated with CWT programs, including bias). Assumptions which need to be met are those which seem to be appropriate. No consensus on methods used to estimate the variance exists. Presently, variance estimation equations are, at best, only approximations and generally inaccurate due to the failure of the tagging and sampling programs to meet the assumptions of large sampling theory or absence of significant covariance terms. Evidence of a lack of agreement between empirical and calculated variances was presented by Webb (1985) and discussed in detail by de Libero (1986). Problems and disagreements concerning proceedures used to presently estimate the contribution and associated variance of a release to a fishery and the design of future tagging and sampling programs will only be resolved when this statistical foundation is established.

The objective of the present study is to develop statistical methodology, appropriate to the Southeastern

Alaska tagging and sampling programs, for the estimation of the contribution of tagged releases to the commercial fisheries and the variance associated with these estimates. The model will employ the more classical approach of developing a probability density function for the probability of recovering a given number of CWTs and deriving expected values for the parameters of interest. Comparison of the variances calculated using this model with those derived by more empirical means is presented. By associating well-studied distributions to each of the sampling events, future tagging and sampling programs can be constructed so as to minimize the sampling variance for a given level of funding. Optimal preseason and inseason allocation of the sampling effort can be realized by projecting the sampling scenario which will minimize the variance of the contribution estimates. Postseason analysis will suggest ways of improving tagging and sampling programs. Although nonsampling errors are not considered in the present model (and somewhat compensated for in the models previously cited) simulation studies may provide adjustment factors for failure to conform to the assumptions. The resulting model will not only provide an estimate of the contribution of tagged releases harvested in Southeastern Alaska commercial fisheries, the variance associated with this estimate, and the covariance between recoveries of different tag codes recovered in the same strata, but may also serve as a guide for improvement in tagging and sampling methods.

CONDUCT OF THE FISHERIES AND DATA COLLECTION

A brief overview of the fisheries and operational plan for CWT sampling in Southeastern Alaska is presented to put the statistical methods in perspective. The Southeastern Region (Region 1) is divided into 25 statistical districts encompassing both inside and offshore waters from Dixon Entrance to Cape Suckling (Fig. 2). Purse seine, gillnet, and troll gear account for over 99% of the commercial harvest of all 5 species of salmon. Fish traps are also used, but are restricted to the Annette Island Fishery Reserve. Purse seine and gillnet harvests occur in discrete areas allowing the catch and sample data to be accurately allocated to specific districts in most cases. Troll catch and sample data are sometimes not attributable to a single statistical district and must be assigned to larger areas which are composed of several districts. Therefore, troll fishery catch and sample data are also reported by Pacific Marine Fisheries Commission (PMFC) area (or nine-area) and by quadrant (or four-area) grouping. Catch and sample data are reported by statistical week, a seven day period beginning at 12:01 AM Sunday and running through 12:00 midnight the following Saturday. As is the case with area strata, purse seine and gillnet harvests are regulated by discrete weekly openings, allowing catch and sample data to be assigned to distinct statistical weeks. However troll deliveries, which may include catches from several statistical weeks are arbitrarily assigned to the last statistical week fished. Therefore, troll sample and catch data are often grouped into multiple time strata. A diagram of the stratification of the catch reporting and sampling program is presented in Figure 3.

During the fishing season samplers are stationed at as many as 20 on-shore delivery sites with traditionally large deliveries and on tenders stationed off-shore. Sampling is conducted on only those boats and tenders whose catch can be assigned to a single area and time stratum. Random sampling of at least 20 percent of the fish harvested by gear type, district, and week is attempted. The sampling data are: port of landing and processor; date sold and date sampled; boat identification; fishing gear; statistical area or areas of harvest; type of sample type (random or select); number of fish sampled (by species) for a missing adipose fin; number of adipose clipped fish counted and marked; the appearance of each adipose clip (good or questionable); and the snout to fork length of each fish lacking an adipose fin. When a salmon without an adipose fin is

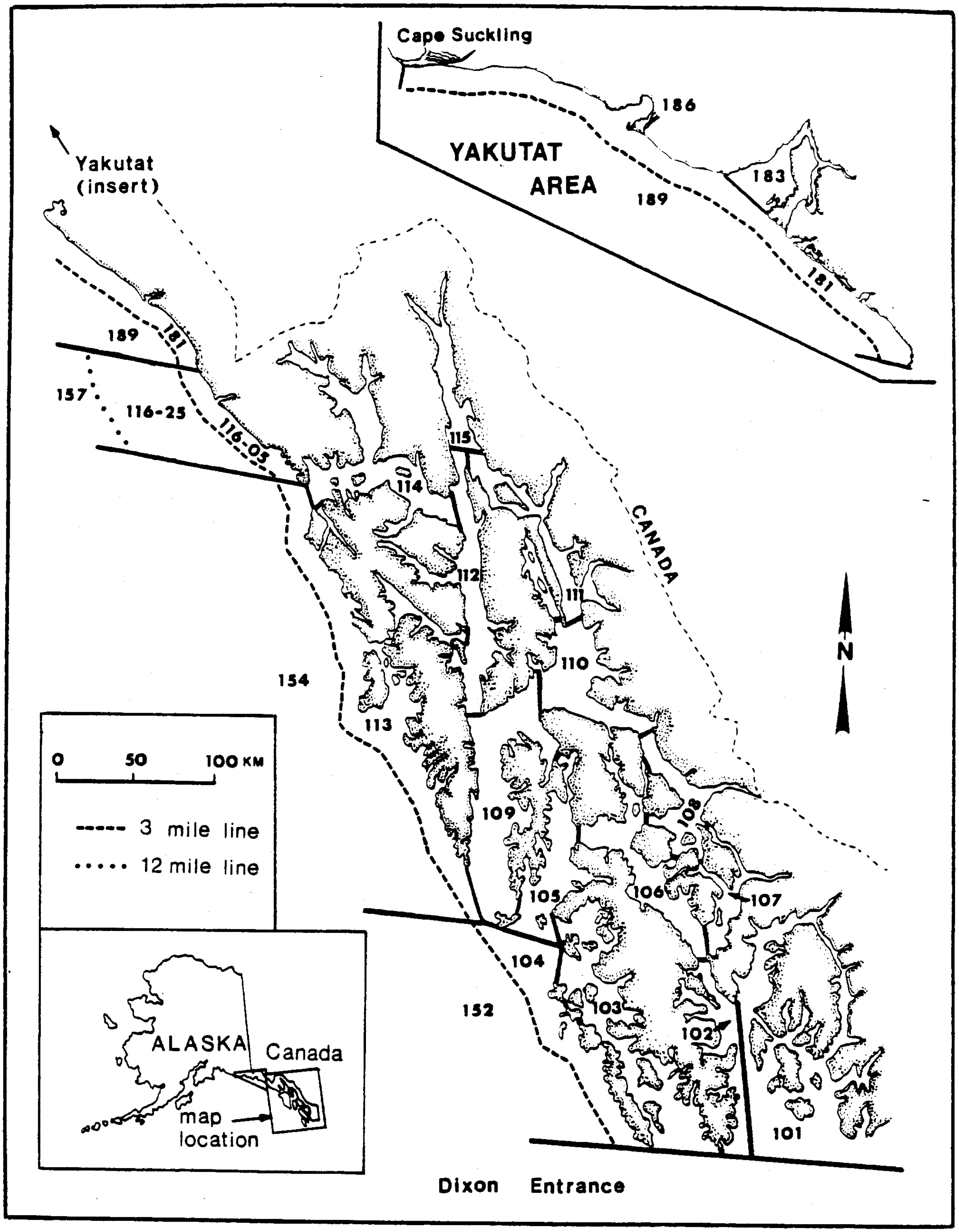


Figure 2. Southeastern Alaska statistical areas.

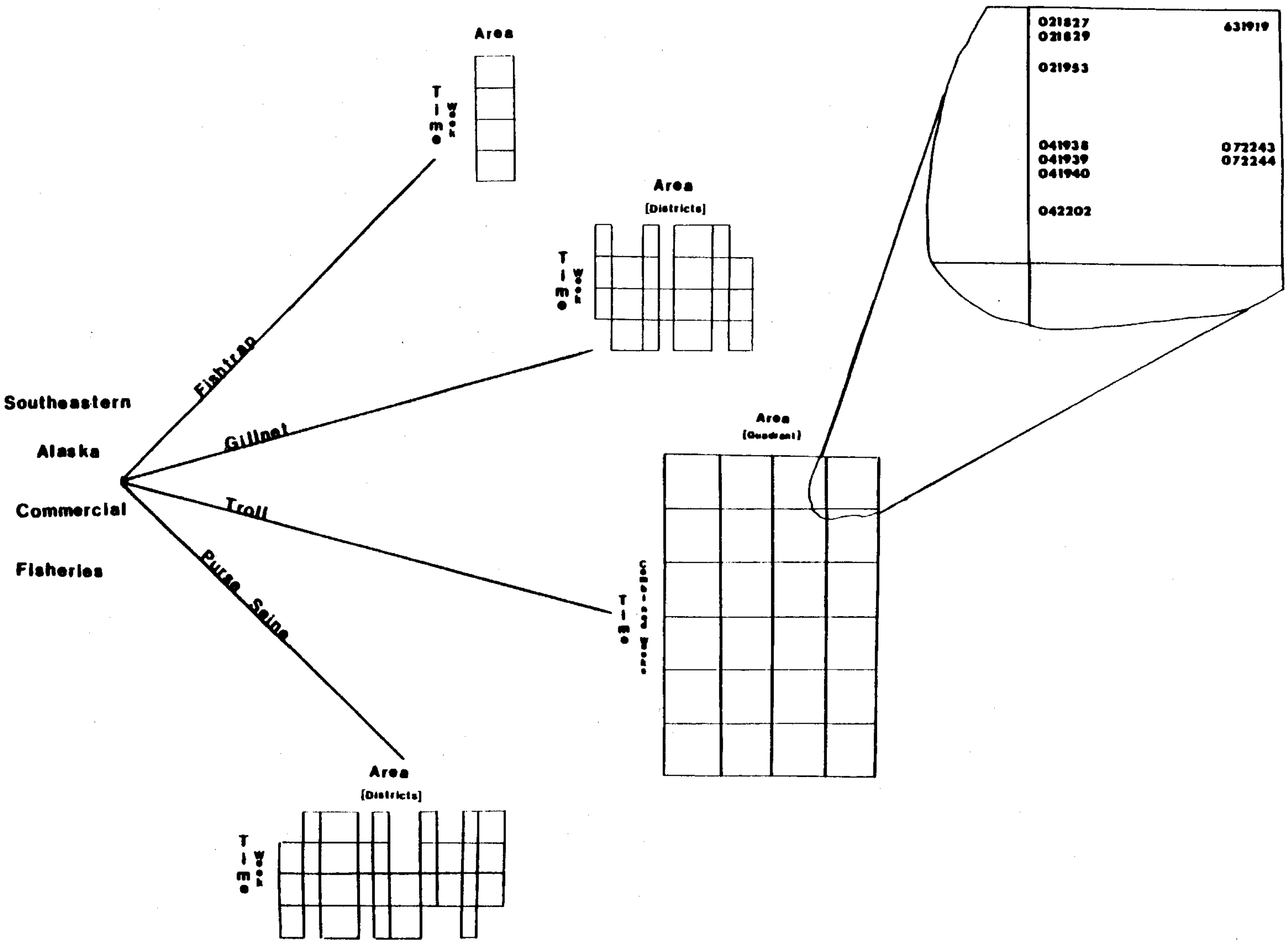


Figure 3. Schematic diagram of fishery-area-time commercial catch strata in Southeastern Ala: Each block represents one statistically independent catch stratum. Within each stratum, coded wire tags of several tag codes may be recovered (enlarged stratum Purse seine and gillnet districts, which are different in size and character of the fisheries, may be opened in different weeks

found, its head is marked with a numbered plastic strap tag before it's processed. Although samplers later attempt to retrieve all marked heads some heads are lost between placement of the head strap and shipment to the head lab. In 1982, 7% of the chinook salmon heads were lost prior to arrival at the tag lab (Clark et al, 1985) and 5% were lost in 1983 (Marshall and Clark, 1986).

At the tag lab, fish heads are examined for the presence of a CWT. If the head containes a CWT, the tag is removed and decoded. A small number of tags are lost after being detected and prior to decoding . In 1982, of the 2,825 tags dissected from the snouts of chinook salmon, 13 were lost (Clark et al; 1985). Loss rates are anticipated to be much smaller in data from later years. In 1983, only 4 of 3,198 tags detected in chinook salmon head samples were lost (Marshall and Clark, 1986).

Commercial catch data are obtained from fish tickets received from fish buyers. For each purchase, a buyer is legally required to record the type of vessel and gear, date of landing, number and pounds of each species, and the statistical area of capture. Data are routinely checked several times for accuracy and completeness. When errors are found which are unresolvable, the catch data are assigned to an unknown time, area, or gear strata.

DEVELOPMENT OF THE STATISTICAL MODEL

By defining the Southeastern Alaska tagging and sampling programs in terms of the probabilities of harvesting, removing from a sample of the commercial catch, and decoding a number of CWTs of a unique tag code, a composite probability density function (PDF) which quantifies the total probability of counting a given number of CWTs in a discrete sampling stratum can be derived. The probability of finding a given number of tags will depend solely upon tagging and sampling information associated with the release group and the time-area-fishery stratum. Applying standard statistical definitions of mathematical expectation (expected value, average, or mean) and variance (second moment about the mean) to the PDF result in equations for the mean number of tags recovered and the variance of this mean number. Modifying the PDF to reflect multiple tag code recoveries in a defined sampling stratum enables us to derive an expression for the covariance (first product moment about the respective means) between tag code recoveries. Although algebraically complex, the derivations are not conceptually difficult. Because of the highly stratified nature of the sampling regime and presence of multiple tag codes in a catch stratum, a large number of symbols are used and the notation may become confusing. A list of the symbols and the variables they represent is provided to facilitate the association of a given variable with the corresponding notation (Table 1.). The variables are also introduced in the text or in the appendix.

The PDF developed will belong to a class of distributions known as compound probability distributions or mixtures (see Kendall and Stuart, 1977, for a discussion of the terminology). Given a distribution function with parameters λ_1, $\lambda_2, \ldots,$ λ_n, a compound distribution is constructed by ascribing to one or more of the parameters a probability distribution. Compound PDF's are generally proposed as more appropriate distributions for statistical analysis of data which do not conform to the assumptions of random sampling (see Johnson and Kotz; 1969). A compound Poisson distribution whose parameter λ is assigned a gamma distribution was demonstrated by Webb (1985) to be a more appropriate model for analysis of British Columbia, Canada, CWT data. We suggest that a

Table 1. Summary of the notation introduced in the text. Coded wire tag is abbreviated to CWT

Symbol	Definition
θ_A	Proportion of a release which contains a CWT of tag code A.
θ_B	Proportion of a release which contains a CWT of tag code B.
θ	Proportion of any release which contains a CWT of a unique tag code.
x_{1_A}	Number of fish in the commercial catch containing a cwt of tag code A.
x_{1_B}	Number of fish in the commercial catch containing a cwt of tag code B.
x_1	Number of fish in the commercial catch containing a cwt of any unique tag code.
x_{2_A}	Number of fish in the commercial catch sample which contain a cwt of tag code A.
x_{2_B}	Number of fish in the commercial catch sample which contain a cwt of tag code B.
x_2	Number of fish in the commercial catch sample which contain a cwt of any unique tag code.
x_{3_A}	Number of fish heads arriving at the tag lab which contain a cwt of tag code A.
x_{3_B}	Number of fish heads arriving at the tag lab which contain a cwt of tag code B.
x_3	Number of fish heads arriving at the tag lab which contain a cwt of any unique tag code.
m_{c_A}	Number of tags dissected out of the fish heads and decoded as tag code A.
m_{c_B}	Number of tags dissected out of the fish heads and decoded as tag code B.
m_c	Number of tags dissected out of the fish heads and decoded as any unique tag code.

Table 1. Summary of the notation introduced in the text (cont.). Coded wire tag is abbreviated to CWT

Symbol	Definition
n_{1_A}	Number of fish in the commercial catch belonging to the release identified by tag code A (tagged and untagged)
n_{1_B}	Number of fish in the commercial catch belonging to the release identified by tag code B (tagged and untagged)
n_1	Number of fish in the commercial catch belonging to the release identified by any unique tag code (tagged and untagged)
n_2	Number of fish in the commercial catch examined for missing adipose fins (commercial catch sample size).
N	Total number of fish in the commercial catch.
a_1	Number of fish missing an adipose fin which are counted by a technician and marked with a head strap.
a_2	Number of fish heads previously marked with a head strap which arrive at the tag lab.
m_1	Number of CWTs which are detected in the fish heads at the tag lab.
m_2	Number of CWTs which are removed from the fish heads and decoded.

Table 1. Summary of the notation introduced in the text (cont.). Coded wire tag is abbreviated to CWT

Symbol	Definition
TC_{n_1}	Total contribution of a release or group of releases to one or more time-area-fishery strata.
r_t	Number of fish of a given release which are coded wire tagged.
r_u	Number of fish of a given release which are not coded wire tagged.
R	Total number of fish in a given release (tagged and untagged).
S	Survival rate
C_{r_t}	Cost of tagging a single smolt.
C_{n_2}	Cost of sampling a single fish.
F_{n_2}	Level of funding allocated to sampling program
F_{r_t}	Level of funding allocated to tagging program
C_T	Total cost of both the tagging and sampling programs.
K_i	Constants defined by products and ratios of given quantities, for i = 1 to 6.
C_i	Constants defined in the appendices for the purpose of simplifying algebraically complex derivations, for i = 1 to 2.
y_{1_x}	Random variable in a general multivariate hypergeometric distribution function (x = A or B)
k_{1_x}	Constants in a general multivariate hypergeometric distribution function (x = A or B)

compound binomial-hypergeometric model is more suitable for Southeastern Alaska CWT data analysis.

The number of CWTs of a given tag code in a catch stratum is dependent upon the number of fish of a given release in the commercial catch and the proportion of that release with CWTs (A release is defined as a group of fish, a known proportion of which have been adipose fin-clipped and contain CWTs of a unique tag code. The remaining fish of the release possess no CWT, although they may be missing an adipose fin.). The probability that a given number of these tags arrive at the tag lab and are decoded is dependent upon the number of fish in the commercial catch examined for missing adipose fins, the number of heads lost before arrival at the decoding lab, and the number of detected tags which are lost prior to decoding. If 100% of the fish from a given release are tagged, all the catch is sampled, and no heads or tags are lost before decoding, then the number of fish of the release in the catch is equal to the number of tags of the corresponding tag code counted at the tag lab. As the proportion of a release tagged and catch sampled decreases, and the number of lost heads and tags increases, the uncertainty in the total number of a release believed to be in the catch increases. Of course, if no fish are tagged, or the catch is not sampled, or all the heads or all the tags are lost, then no estimate can be made on the number of a release in the catch.

A group of immature fish, of which a known proportion (Θ) is coded wire tagged (CWTed), is released. After an interval of time (usually 1 or more years), a relatively small number of those fish have survived and are recruited into the commercial fisheries. Of the number of fish of this release which are commercially caught in a discrete time-area-fishery stratum (n_1), some have CWTs (x_1) and some do not, resembling natural or untagged hatchery stocks (n_1 - x_1 fish). Because tagged and untagged fish are assumed to have the same probablility of survival and capture, the probability of a harvested fish of the release containing a CWT is Θ, the proportion of the release CWTed. The number of fish of the release caught in one or more strata of interest is generally small, relative to the total number of live fish at time of release (generally less than 5%. A large percentage of hatchery released salmonids suffer some type of mortality before being recruited into the fishery. The remaining fish will either be caught or will escape back to the release site). The removal of each individual fish by the

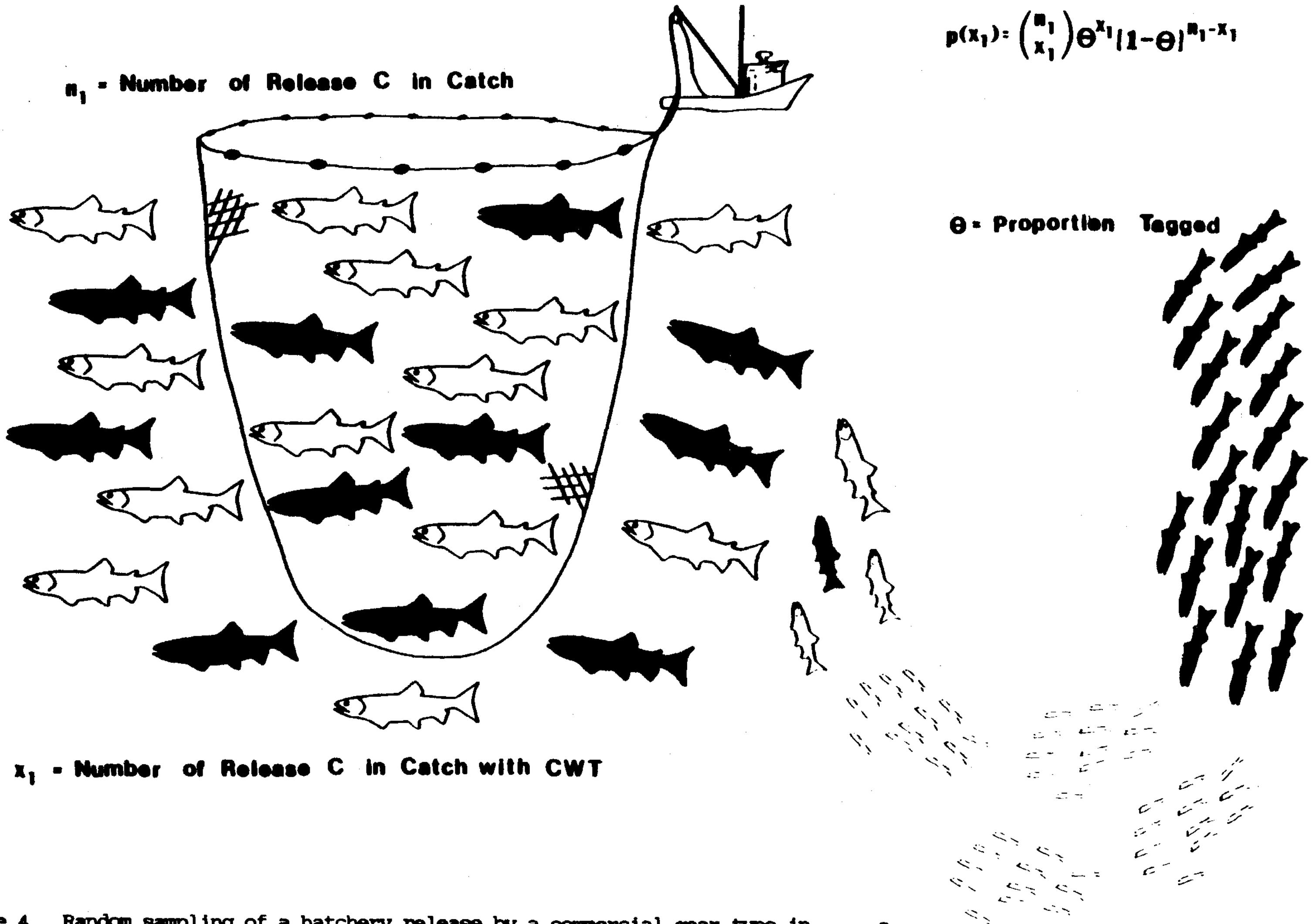


Figure 4. Random sampling of a hatchery release by a commercial gear type in a defined catch stratum containing tagged (dark, adiposeless) and untagged (dark) fish from a given release, tagged (light, adiposeless) and untagged (light) fish from other releases, and wild stock (light) fish. The number of tagged fish of a given release caught by the fleet is assumed to be binomially distributed.

commercial fishery from the total number of fish in the release is therefore considered to be an independent event and the random variable, x_1 is assumed to be binomially distributed (Fig. 4) such that the probability of harvesting x_1 CWTed fish among the n_1 total number of fish of the release in the catch is:

$$p(x_1) = \binom{n_1}{x_1} \Theta^{x_1}(1-\Theta)^{n_1-x_1} \qquad [1]$$

where:

$$\binom{n_1}{x_1} = \frac{n_1!}{x_1!\ (n_1-x_1)!} \qquad [2]$$

If we examine the entire catch, remove the head of each clipped fish, and remove and identify the release of each CWT, the probability of counting x_1 CWTs is given by Equation 1. However, we are seldom able to examine all of the commercial catch, and the number of CWTs recovered is also dependent upon the fraction of the catch sampled for adipose fin-clipped fish. Given that there are a total of x_1 CWTed fish in a commercial catch of known size N, n_2 fish in the commercial catch are examined for missing adipose fins, and all adipose clipped fish are recognized and heads marked for later retrival, the probability of finding x_2 CWTed fish in the sample of n_2 fish is described by a hypergeometric distribution, conditional on x_1 CWTed fish being present in the commercial catch (Fig. 5):

$$p(x_2|x_1) = \frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \qquad [3]$$

A small fraction (generally less than 10%) of the heads marked as containing CWTs are lost before arriving at the head dissection and tag decoding lab. Because of the likely presence of CWTs of more than one tag code in the

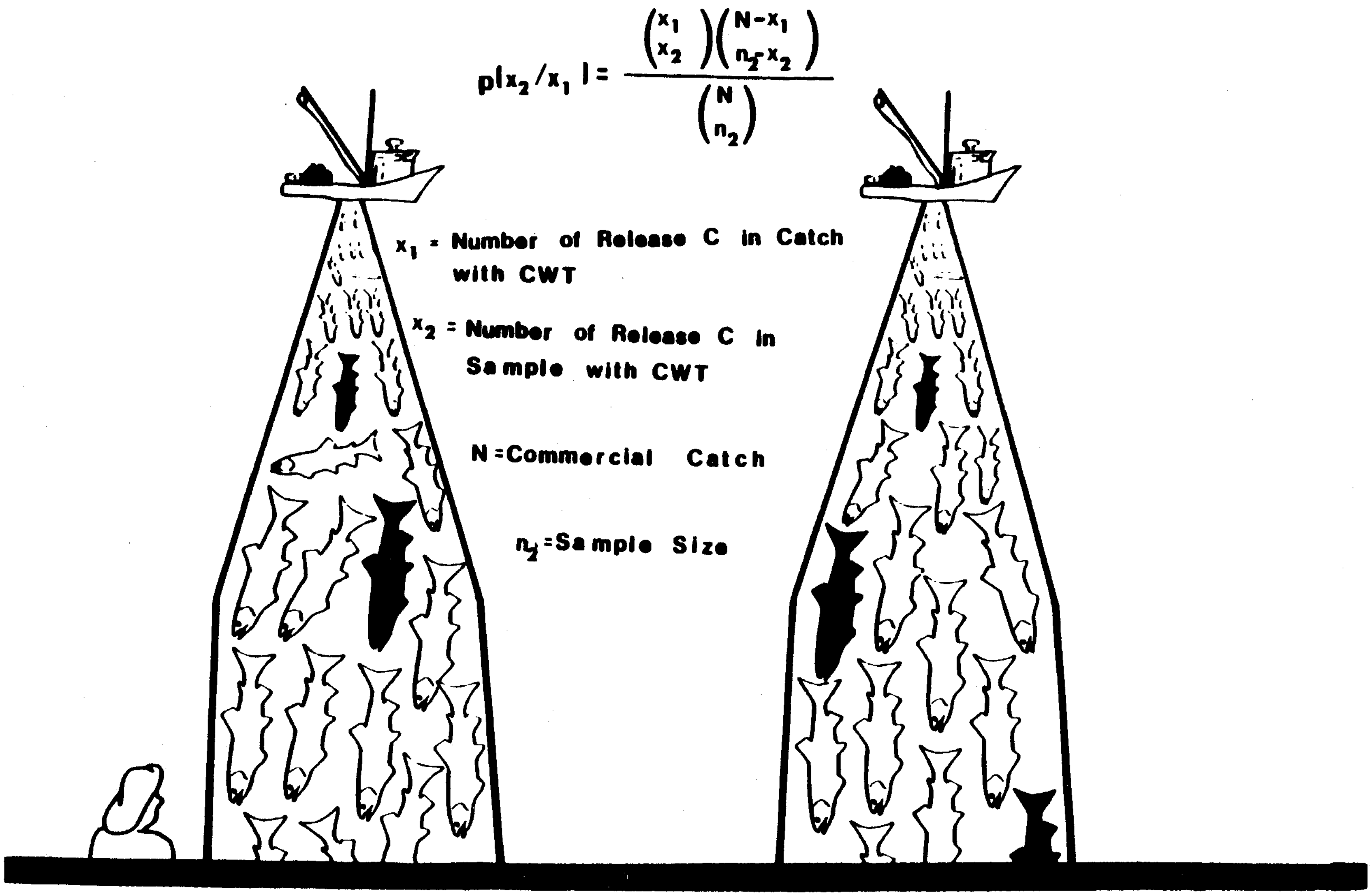


Figure 5. Random sampling of the commercial catch. The number of tagged fish of a given release (dark, adiposeless fish) counted by a technican examining a proportion of the commercial catch assumed to be hypergeometrically distributed, conditional on the total number of tagged fish of the given release in the catch.

sample, or of fish missing an adipose fin but possessing no CWT, allocation of tag codes to the unrecovered CWTs in the lost heads involves some measure of uncertainty. Given that a total of a_1 fish without adipose fins were counted and their heads marked for later recovery, of which x_2 fish contained a CWT of a release of interest (the probability of x_2 tags being in the sample is given by Equation 3), the number of fish in the commercial catch sample which contain CWTs identifying a particular release is known if the heads of all marked fish arrive at the tag lab, and CWTs are removed and decoded for all fish heads that contain a tag. However, if 1 or more heads are lost and the probability of losing any given head in the stratum is equal to the probability of losing any other head, the probability distribution of the number of heads which arrive at the tag lab (a_2) is analogous to that describing the number of heads in a sample which is removed from the total population of marked heads at the processor (Figure 6). The probability of x_3 heads containing a CWT of a given tag code in a sample size of a_2 , given a total population size of a_1 (the difference between a_1 and a_2 being the number of lost heads), is described by a hypergeometric distribution conditional on x_2 CWTs of the given tag code in a_1:

[4]

$$p(x_3|x_2) = \frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}}$$

The distribution of tag codes, recovered from tags dissected out and decoded in the lab, over lost tags (defined as tags which were detected in fish heads, but lost prior to decoding) is approached in a manner similar to the allocation of tag codes to lost heads (Figure 6). Given that m_1 CWTs arrive at the tag lab and are detected, of which x_3 are of a given tag code; and only m_2 tags are successfully removed and decoded (the difference, $m_1 - m_2$ being the number of lost tags); the probability of counting m_c decoded tags of the given tag code, conditional on x_3 tags arriving at the tag lab, is

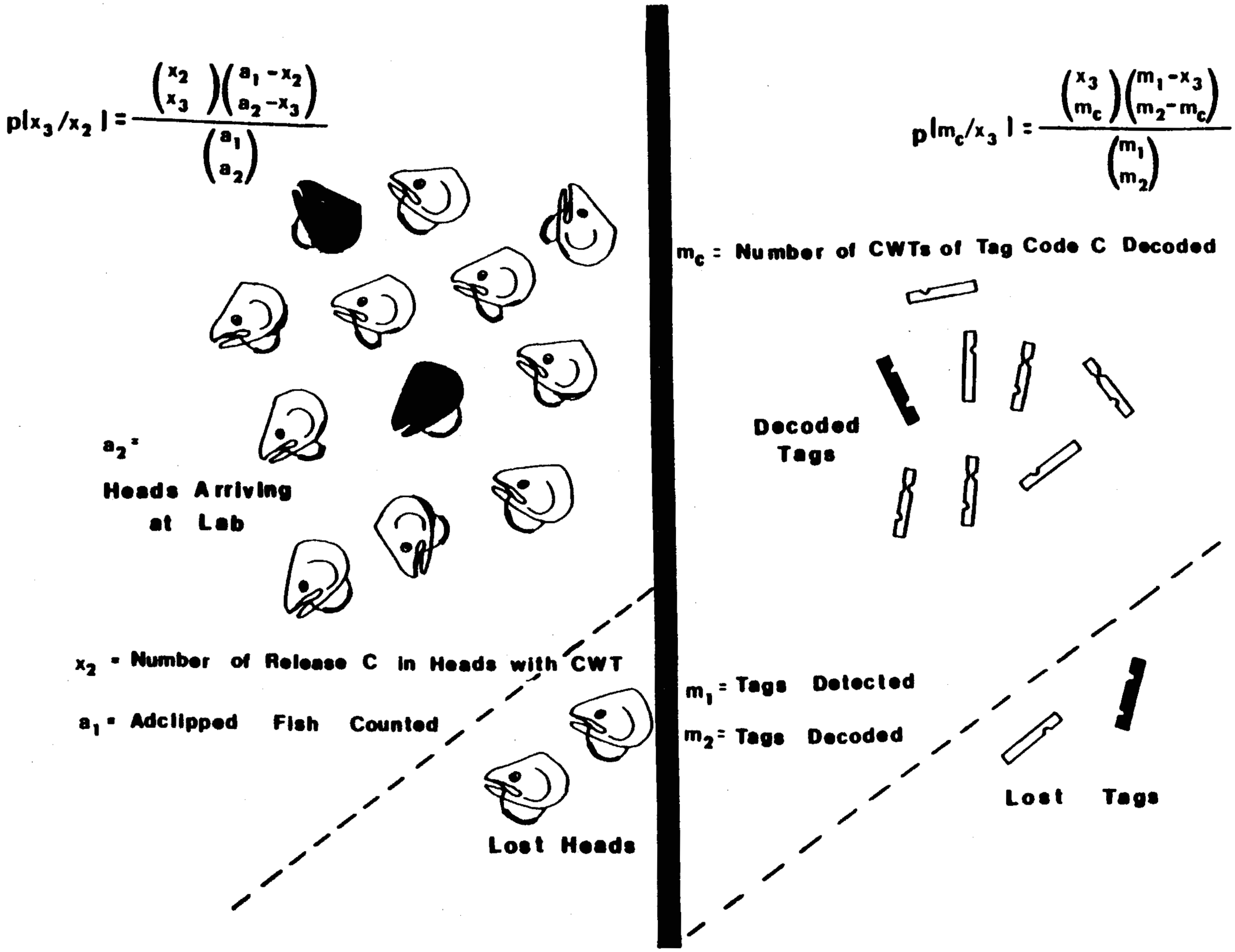


Figure 6. Random loss of fish heads and coded wire tags. The number of heads containing a tag identifying a given release (dark heads) arriving at the tag lab and the number of tags of a given tag code (dark tags) removed and decoded at the tag lab are both assumed to be hypergeometrically distributed.

$$p(m_c | x_3) = \frac{\binom{x_3}{m_c}\binom{m_1 - x_3}{m_2 - m_c}}{\binom{m_1}{m_2}} \quad [5]$$

By Bayes' Theorem, the joint probability distribution function which quantifies the likelihood of x_1, x_2, x_3, and m_c occuring in the sampling event is the product of each conditional probability (Equations 1, 3, 4, and 5):

$$p(x_1;\ x_2;\ x_3;\ m_c) = \binom{n_1}{x_1}\Theta^{x_1}(1-\Theta)^{n_1-x_1}\left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}}\right]\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}}\right]\left[\frac{\binom{x_3}{m_c}\binom{m_1-x_3}{m_2-m_c}}{\binom{m_1}{m_2}}\right] \quad [6]$$

The probability that m_c CWTed fish of a given release are harvested by a commercial fishery in a defined time-area stratum; identified by a sampler examining a fraction of the catch; arriving at the tag lab; and dissected out and decoded is the sum of all possible sampling senarios which result in m_c. Mathematically, this is defined as the marginal distribution of m_c for all possible values of x_1, x_2, and x_3:

[7]

$$p(m_c) = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} (1-\Theta)^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] \left[\frac{\binom{x_3}{m_c}\binom{m_1-x_3}{m_2-m_c}}{\binom{m_1}{m_2}} \right]$$

Derivation of an expression for the expected value of m_c is achieved by evaluating the sum:

[8]

$$E(m_c) = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} (1-\Theta)^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] \sum_{m_c=0}^{x_3} m_c \left[\frac{\binom{x_3}{m_c}\binom{m_1-x_3}{m_2-m_c}}{\binom{m_1}{m_2}} \right]$$

Appendix A presents a detailed outline of the derivation of the expected value of m_c (Equation A12).

[9]

$$E(m_c) = \left(\frac{m_2}{m_1}\right) \left(\frac{a_2}{a_1}\right) \left(\frac{n_2}{N}\right) \Theta \; n_1$$

An unbiased estimate of n_1 results (Equation A13):

[10]

$$E(n_1) = \hat{n}_1 = \left(\frac{m_1}{m_2}\right) \left(\frac{a_1}{a_2}\right) \left(\frac{N}{n_2}\right) \frac{m_c}{\Theta}$$

The equation which generates the variance of m_c, denoted by $Var(m_c)$ is derived by using the relationship:

[11]

$$Var(m_c) = E[m_c(m_c-1)] + E[m_c] - (E[m_c])^2$$

and evaluating $E[m_c\ (m_c\ -1)]$ (Appendix B). The variance of m_c is (Equation B18):

[12]

$$\mathrm{Var}(m_c) = \frac{(m_2)(m_2-1)(a_2)(a_2-1)(n_2)(n_2-1)(n_1)(n_1-1)\theta^2}{(m_1)(m_1-1)(a_1)(a_1-1)(N)(N-1)}$$

$$+ \frac{m_2 a_2 n_2 n_1 \theta}{m_1 a_1 N} - \left(\frac{m_2 a_2 n_2 n_1 \theta}{m_1 a_1 N}\right)^2$$

The variance of $\hat{n}_1$, $\mathrm{Var}(\hat{n}_1)$ is (Equation B21):

[13]

$$\mathrm{Var}(\hat{n}_1) = \frac{(m_1)(m_2-1)(a_1)(a_2-1)(N)(n_2-1)(n_1)(n_1-1)}{(m_2)(m_1-1)(a_2)(a_1-1)(n_2)(N-1)}$$

$$+ \frac{m_1 a_1 N n_1}{m_2 a_2 n_2 \theta} - n_1^2$$

and an estimate of the variance of $\hat{n}_1$, $S^2(\hat{n}_1)$, is (Equation B22):

[14]

$$S^2(\hat{n}_1) = \left[m_c \left(\frac{N m_1 a_1}{m_2 a_2 n_2 \Theta} \right)^2 \right]$$

$$\left[1 - m_c + \left(\frac{(m_2-1)(a_2-1)(n_2-1)}{(m_1-1)(a_1-1)(N-1)} \right) \left(\frac{m_1 a_1 N m_c}{m_2 a_2 n_2} - \Theta \right) \right]$$

Appendix C demonstrates that $S^2(\hat{n}_1)$ is biased, such that an unbiased estimated of $Var(\hat{n}_1)$ is:

[15]

$$E[Var(\hat{n}_1)] =$$

$$\left[\left(\frac{m_2}{m_2-1} \right) \left(\frac{m_1-1}{m_1} \right) \left(\frac{a_2}{a_2-1} \right) \left(\frac{a_1-1}{a_1} \right) \left(\frac{n_2}{n_2-1} \right) \left(\frac{N-1}{N} \right) \right] S^2(\hat{n}_1)$$

Equations 10 and 14 enable us to estimate the number of fish of a given release in defined time-area-fishery stratum, and the variance associated with this estimate. However, the contribution of interest is often the sum of the estimated number of fish of the release in many strata; the total number of fish of two or more releases harvested in a given time-area-fishery stratum; or the total contribution of two or more releases to several commercial catch strata. An unbiased estimate of the contribution (TC_{n_1}) of releases represented by 1 or more tag codes to the commercial catches of 1 or more sampling strata is the sum of the estimated contributions of each individual release in each independent sampling strata:

[16]

$$TC_{n_1} = \sum_{i=1}^{s} \sum_{j=1}^{t} \hat{n}_{1_{ij}}$$

for i = 1 to s strata and j = 1 to t tag codes. Estimation of the variance associated with this sum assumes each sampling strata be statistically independent of other sampling strata (see Fig. 3);

[17]

$$Var(TC_{n_1}) = \sum_{i=1}^{s} \sum_{j=1}^{t} Var(\hat{n}_1)_{ij} + 2 \sum_{i=1}^{s} \sum_{j=1}^{t} \sum_{k>j}^{t} Cov(\hat{n}_{1j};\hat{n}_{1k})$$

Calculation of the variance associated with the estimated contribution of two or more releases to a single commercial catch stratum requires that statistical relationships be developed to estimate the covariance between expected values of the tag recoveries of any two releases. The PDF's presented in Equations 1, 3, 4, and 5 are expanded by replacing the limited, univariate hypergeometric distributions by the more general, multivariate hypergeometric distributions. It should be noted that the univariate hypergeometric distribution is a special case of the multivariate form, the only difference is that the multivariate form of the distribution represents the joint probability of realizing two or more outcomes.

A modification of the notation used in Equations 1 - 15 is necessary in order to specify to which of two different tag codes we are referring. Therefore, for tag code A, let:

Θ_A be the fraction of released fish which contain a CWT of tag code A.

n_{1_A} be the number of fish of the release group represented by tag code A (both tagged and untagged) which are harvested by the commercial fleet in a given time-area-fishery stratum.

x_{1_A} be the number of fish containing a CWT of tag code A in the commercial catch.

x_{2_A} be the number of fish containing a CWT of tag code A in the sample from the commercial catch.

x_{3_A} be the number of fish containing a CWT of tag code A in the heads which arrive at the tag lab.

m_{c_A} be the number of tags of tag code A which are decoded.

and for tag code B, the corresponding variables are:

Θ_B the fraction of released fish which contain a CWT of tag code B.

n_{1_B} the number of fish of the release group represented by tag code B (both tagged and untagged) which are harvested by the commercial fleet in a given time-area-fishery stratum.

x_{1_B} the number of fish containing a CWT of tag code B in the commercial catch.

x_{2_B} the number of fish containing a CWT of tag code B in the sample from the commercial catch.

x_{3_B} the number of fish containing a CWT of tag code B in the heads which arrive at the tag lab.

m_{c_B} the number of tags of tag code B which are decoded.

Given that there are x_{1_A} fish which contain a CWT of tag code A in the catch, x_{1_B} fish which contain a CWT of tag code B in the catch, and $N - x_{1_A} - x_{1_B}$ fish which either do not contain a CWT or contain a CWT of a different tag code, the multivariate hypergeometric distribution analogous to Equation 3 is:

$$p(x_{2_A};x_{2_B}|x_{1_A};x_{1_B}) = \frac{\binom{x_{1_A}}{x_{2_A}}\binom{x_{1_B}}{x_{2_B}}\binom{N-x_{1_A}-x_{1_B}}{n_2-x_{2_A}-x_{2_B}}}{\binom{N}{n_2}} \qquad [18]$$

The multivariate form of Equations 4 and 5 are respectively:

$$p(x_{3_A};x_{3_B}|x_{2_A};x_{2_B}) = \frac{\binom{x_{2_A}}{x_{3_A}}\binom{x_{2_B}}{x_{3_B}}\binom{a_1-x_{2_A}-x_{2_B}}{a_2-x_{3_A}-x_{3_B}}}{\binom{a_1}{a_2}} \qquad [19]$$

and

$$p(m_{c_A};m_{c_B}|x_{3_A};x_{3_B}) = \frac{\binom{x_{3_A}}{m_{c_A}}\binom{x_{3_B}}{m_{c_B}}\binom{m_1-x_{3_A}-x_{3_B}}{m_2-m_{c_A}-m_{c_B}}}{\binom{m_1}{m_2}} \qquad [20]$$

The multivariate compound distribution representing the probability of counting m_{c_A} and m_{c_B} tags in the catch sample is:

[21]

$$p(m_{c_A};m_{c_B}) = \sum_{x_{1_A}=0}^{n_{1_A}} \sum_{x_{1_B}=0}^{n_{1_B}} \sum_{x_{2_A}=0}^{x_{1_A}} \sum_{x_{2_B}=0}^{x_{1_B}} \sum_{x_{3_A}=0}^{x_{2_A}} \sum_{x_{3_B}=0}^{x_{2_B}} \binom{n_{1_A}}{x_{1_A}} \Theta_A^{\,x_{1_A}} (1-\Theta_A)^{n_{1_A}-x_{1_A}}$$

$$\binom{n_{1_B}}{x_{1_B}} \Theta_B^{\,x_{1_B}} (1-\Theta_B)^{n_{1_B}-x_{1_B}} \quad \frac{\binom{x_{1_A}}{x_{2_A}} \binom{x_{1_B}}{x_{2_B}} \binom{N-x_{1_A}-x_{1_B}}{n_2-x_{2_A}-x_{2_B}}}{\binom{N}{n_2}}$$

$$\frac{\binom{x_{2_A}}{x_{3_A}} \binom{x_{2_B}}{x_{3_B}} \binom{a_1-x_{2_A}-x_{2_B}}{a_2-x_{3_A}-x_{3_B}}}{\binom{a_1}{a_2}} \quad \frac{\binom{x_{3_A}}{m_{c_A}} \binom{x_{3_B}}{m_{c_B}} \binom{m_1-x_{3_A}-x_{3_B}}{m_2-m_{c_A}-m_{c_B}}}{\binom{m_1}{m_2}}$$

An equation which estimates the covariance of n_{1_A} and n_{1_B} is derived in Appendix D:

$$\mathrm{Cov}(n_{1_A};n_{1_B}) = \hat{n}_{1_A}\hat{n}_{1_B}\left[\frac{m_1(m_2-1)a_1(a_2-1)N(n_2-1)}{m_2(m_1-1)a_2(a_1-1)n_2(N-1)} - 1\right] \qquad [22]$$

where $\hat{n}_{1_A}$ and $\hat{n}_{1_B}$ are defined by Equation 10.

VERIFICATION OF THE MODEL

A practical concern of the Southeastern Alaska tagging and sampling programs is whether the assumptions implicit in the statistical methodology are met. Careful and complete examination of these programs would be necessary to detect specific deviations from these assumptions. Tagging operations need to be monitored to verify random selection of juvenile fish to be tagged, equal probability of survival of tagged and untagged fish, identical treatment of tagged and untagged fish, and minimal tag loss. Sampling programs need to be inspected for random and independent sampling, head loss, and tag loss in all catch strata; correct identification of tag codes; recognition of all fish missing adipose fins in a sample; and accurate catch and sample data. The complexity of most sampling programs, increasing numbers of hatchery releases, and fiscal limitations generally preclude this type of evaluation. However, a more general means of comparing variances estimated by the multivariate compound binominal-hypergeometric (MCBH) model with empirically estimated variances can be devised by replicate tagging of hatchery releases.

Replicate tagging has been employed in Alaskan hatcheries to study the effects of different practices on the timing, survival, growth, and other biological attributes of hatchery-reared salmon. Tagging programs to measure the variability of contribution estimates are presently being implemented and adult salmon from these tagged releases will be returning in two years. Although there are as yet no replication data from Alaska with which to compare the results of the MCBH model, five releases from Whitman Lake Hatchery in 1982 and 1983 were tagged in a manner similar to replicate tagging (Gary Freitag, pers. comm.). Due to insufficient numbers of CWTs of a single code, two releases of chinook salmon and three releases of coho salmon were tagged with two different codes. Rearing and release treatments were identical for salmon containing both codes within each of the five releases (see Table 2 for a list of the tag codes). Adult returns of the five groups were harvested in the 1985 commercial fisheries in relatively large numbers and in many of the time-area-fishery strata.

Unfortunately, direct calculation of the variation associated with the number of tag recoveries and with the estimated contribution of a release which has been tagged

Table 2. Whitman Lake Hatchery releases which were coded wire tagged with two tag codes representing replicate tagging.

Species	Stock	Age at Return in 1985	Release Site	Release Date	Tag Code	Number Tagged	Number Untagged	Proportion Tagged
Chinook	Cripple Creek	1.2	Neets Bay	May, 1983	B40907	94,723	1,933	0.701
					B40908	37,737	770	0.279
Chinook	Cripple Creek	1.1	Neets Bay	June, 1984	042430	10,335	25,714	0.141
					042431	10,792	26,699	0.147
Coho	Whitman Lake	1.1	Herring Cove	May, 1984	042240	10,454	11,773	0.225
					042256	10,020	12,253	0.235
Coho	Neets Bay	1.1	Neets Bay	June, 1984	042257	10,349	155,573	0.032
					042258	10,159	152,478	0.031
Coho	Neets Bay	1.1	Neets Bay	June, 1984	042259	10,774	303,865	0.017
					042260	10,630	304,010	0.017

with two or more codes is not possible if different numbers of fish represented by each code have been tagged. Due to differing probabilities

of sampling a given number tagged fish of each code, empirically obtained variances are approximated only by complicated, weighted formulas. As was demonstrated in the previous section, the number of CWT recoveries of each code is not independent of the recoveries of other codes within the same sampling stratum. In addition, because the variance associated with the number of CWT recoveries is a function of the proportion of a release tagged and the number of fish of the release in the commercial catch, the number of CWT recoveries are identically distributed only if an equal number of fish are tagged with each code. Therefore, empirically estimating variances associated with CWT recoveries by averaging the squared deviation of the number of CWT recoveries of each code from the average number of recoveries will provide statistically inconsistent estimates of the variance. The variance of the difference between the estimated contributions of a release using two different codes is a more correct comparison.

A separate estimate of n_1 was made using each tag code from the Whitman Lake Hatchery, and the difference between these estimates was used to calculate a variance to compare with that calculated from the MCBH mode. It is intuitively obvious and can easily be shown that the expected value of the difference between two estimated contributions of a given release using tag recoveries from two replicate tag codes is 0:

[23]

$$E\left[\hat{n}_{1_A} - \hat{n}_{1_B}\right] = 0$$

The variance of the difference is:

[24]

$$\mathrm{Var}\left[\hat{n}_{1_A} - \hat{n}_{1_B}\right] = \mathrm{Var}\left[\hat{n}_{1_A}\right] + \mathrm{Var}\left[\hat{n}_{1_B}\right] - 2\mathrm{Cov}\left[\hat{n}_{1_A}; \hat{n}_{1_B}\right]$$

Equation 15 was used to calculate $\mathrm{Var}\left[\hat{n}_{1_A}\right]$ and $\mathrm{Var}\left[\hat{n}_{1_B}\right]$ and Equation 22 to calculate $\mathrm{Cov}\left[\hat{n}_{1_A};\ \hat{n}_{1_B}\right]$, and the results were substituted into Equation 24 to give an estimate of $\mathrm{Var}\left[\hat{n}_{1_A} - \hat{n}_{1_B}\right]$ from the MCBH model.
An empirical estimate of $\mathrm{Var}\left[\hat{n}_{1_A} - \hat{n}_{1_B}\right]$ is derived from the difference itself. It can be shown that:

$$E\left[\left(\hat{n}_{1_A} - \hat{n}_{1_B}\right)^2\right] = \mathrm{Var}\left(\hat{n}_{1_A}\right) + \mathrm{Var}\left(\hat{n}_{1_B}\right) - 2\mathrm{Cov}\left(\hat{n}_{1_A};\hat{n}_{1_B}\right) \qquad [25]$$

Note that the right-hand sides of Equations 24 and 25 are identical, which makes the square of the difference an estimate of the variance of the difference.

Comparison of the empirical variances and MCBH model variances are presented in detail in Appendix E Tables E1 - E9 and summarized in Tables 3 and 4. Variances are calculated for each time-area-gear stratum. Due to the more discrete nature of the conduct of net fisheries when compared to the conduct of the troll fishery, these fisheries are considered separately. Differences between one or two large variances within a species and fishery tend to conceal differences between other smaller variances. Therefore, the 'coefficient of variance' is a better quantity for interstrata comparisons. The coefficient of variance is calculated as the square root of the variance divided by the estimated contribution of combined tag codes and is analogous to but not equal to the coefficient of variation.

Empirical variances were not consistently different from variances calculated using the MCBH model. Of the 9 comparisons, 6 of average empirical variances were larger than the average MCBH variance (Table 3). However, only 2 of the average empirical coefficients of variance were larger than the average MCBH coefficients of variance. Of the 227 time-area-gear-release comparisons made, 129 of the empirical variances and 98 of the MCBH variances were larger. The difference in the distributions of the variances in all 8 comparisons (tag releases 042430 and 042431 were not compared due to the small number of strata), the MCBH variances were larger than the empirical variances in strata with large variances or large absolute differences between variances (of the 10 strata with the

Table 3. Comparison of the Empirical and MCBH variances and coefficients of variance. Strata with larger variances are the number of strata of a fishery and paired tag codes which have the larger of the two variances calculated by MCBH and empirical methods.

Tag Codes	Species	Fishery	Number of Strata	Average Variance of Difference		Average Coefficient of Variance		Strata with Larger Variances	
				MCBH	Empirical	MCBH	Empirical	MCBH	Empirical
B40907/B40908	Chinook	Troll	29	864	618	0.634	0.583	18	11
B40907/B40908	Chinook	Net	34	72	73	1.368	1.449	10	24
042430/042431	Chinook	Net	7	382	425	1.800	1.980	0	7
042240/042256	Coho	Troll	18	12,619	22,635	1.075	0.858	9	9
042240/042256	Coho	Net	42	5,193	6,407	1.178	1.029	20	22
042257/042258	Coho	Troll	17	1,316,196	1,462,371	0.862	0.789	9	8
042257/042258	Coho	Net	36	91,269	129,560	1.290	1.264	13	23
042259/042260	Coho	Troll	17	3,006,358	1,204,786	1.105	0.863	9	8
042259/042260	Coho	Net	27	182,756	145,623	1.464	1.170	10	17

largest differences in variances in each comparison, the MCBH variance was larger in 5 to 10 of these strata for the 8 comparisons). The empirical variances were larger in strata with small variances or small absolute differences between the variances (of the strata with the 10 smallest differences in variances, the empirical variance was larger in 5 to 10 of these strata for the 8 comparisons).

Fundamental differences in the underlying probability distributions of MCBH and empirical variances preclude the use of parametric and nonparametric statistical tests to discern if significant differences exist between the variances (refer to Conover, 1980, for a list of the assumptions which need to be met in order to use a sign test or Wilcoxon test to test for differences between means. The variances are obviously not internally consistent or their differences symmetrically distributed).

No consistent trends in the differences between the variances were found in the comparisons of chinook and coho salmon catches. Due to the small proportion of coho salmon tagged, both empirical and MCBH variances were larger than those of chinook salmon, but the relative differences between empirical and MCBH variances were similar for both species. The average empirical variances were larger in 2 of the 3 chinook salmon comparisons and in 4 of the 6 coho salmon comparisons. The average MCBH coefficients of variance were larger in 1 of the 3 chinook salmon comparisons and in all of the coho salmon comparisons. The empirical variance of the net fisheries tended to be larger than the corresponding MCBH variances. These differences were especially large for the 042430 and 042431 chinook salmon recoveries (all 7 empirical variances were larger than the corresponding MCBH variances) and the 042257 and 042258 coho salmon recoveries. However, the uncertainty of whether the MCBH model underestimates the variance of contribution estimates in the net fisheries remains (the average MCBH variance was larger than the average empirical variance of 1 coho salmon net fisheries comparison). Future recoveries of replicate tag releases may reveal areas of improvement in the sampling of these fisheries.

A more relevant comparison in some applications is between the estimated accuracy of the total estimated contribution of a release to all strata and the observed variation in estimates of the same contribution using recoveries of each different code (Table 4). A nonquantitative

Table 4. Comparison of the number of recoveries and the total estimates of the contribution of a release using recoveries of different replicate tag codes. Tag codes are arbitrarily assigned to Code A or Code B. Combined estimates are the total contributions estimated by considering codes A and B as a single tag code.

Tag Codes: Code A	Tag Codes: Code B	Species	Fishery	Number of Strata	Number of CWT Recovered: Code A	Number of CWT Recovered: Code B	Ratio Code A:B at Recovery	Ratio Code A:B at Release
B40907	B40908	Chinook	Troll	29	312	101	0.755	0.715
B40907	B40908	Chinook	Net	34	63	22	0.741	0.715
042430	042431	Chinook	Net	7	2	6	0.250	0.489
042240	042256	Coho	Troll	18	85	85	0.500	0.489
042240	042256	Coho	Net	42	144	151	0.488	0.489
042257	042258	Coho	Troll	17	123	125	0.496	0.505
042257	042258	Coho	Net	36	97	97	0.500	0.505
042259	042260	Coho	Troll	17	48	53	0.475	0.503
042259	042260	Coho	Net	27	34	49	0.410	0.503

Tag Codes: Code A	Tag Codes: Code B	Species	Fishery	Estimated Contribution and Approximate 95% confidence intervals: Code A	Code B	Combined
B40907	B40908	Chinook	Troll	1,370 +/- 191	1,020 +/- 243	1,269
B40907	B40908	Chinook	Net	224 +/- 71	161 +/- 65	205
042430	042431	Chinook	Net	30 +/- 43	88 +/- 29	59
042240	042256	Coho	Troll	1,596 +/- 336	2,208 +/- 871	1,895
042240	042256	Coho	Net	2,045 +/- 827	2,198 +/- 391	2,121
042257	042258	Coho	Troll	24,005 +/- 7,713	19,161 +/- 5,138	21,606
042257	042258	Coho	Net	9,814 +/- 2,162	12,078 +/- 2,797	10,935
042259	042260	Coho	Troll	16,607 +/- 8,332	20,402 +/- 11,261	18,492
042259	042260	Coho	Net	7,052 +/- 2,787	10,287 +/- 3,330	8,658

criterion of comparison is the relative magnitude of the difference between the contribution estimates of a release using recoveries of different codes compared to the estimated precision of each contribution estimate. The 95% confidence limits were calculated by multiplying the square root of the variances by 1.96. These confidence limits must be considered approximate because of the highly skewed nature of the distribution of the estimate when few tags are recovered. The observed variability in contribution estimates compares well with that predicted by the MCBH model. The contribution estimates of the five releases using combined tag recoveries were outside the 95% confidence limits of the estimated contributions using recoveries of only one code in one comparison out of 18. Six estimates using recoveries of one tag code were outside the 95% confidence interval of the estimates using recoveries of the other tag code in 18 comparisons.

TAGGING AND SAMPLING RATES

Funding, personnel, and logistic limitations generally preclude 100% tagging and sampling rates. Therefore, the planners of CWT programs need to determine the minimum tagging and sampling rates necessary to obtain a desired level of precision in contribution estimates. Determination of minimum tagging and sampling rates requires an understanding of the relationship between the proportions of releases tagged, the numbers of fish expected to be harvested, the proportion of catch which is sampled, and the variance of contribution estimates. In sampling programs in which relatively large numbers of heads of coded wire tagged fish are lost before arrival at the tag lab or large numbers of detected tags are lost before decoding, the varinace will also depend on the magnitude of these losses. Stratification of the sampling program, multiple releases represented by 2 or more tag codes, and non-sampling errors also contribute to the variance.

The variance decreases as more fish are tagged and as more fish are sampled, and increases as more fish of the release contribute to the catch. The number of fish sampled (n_2) and the proportion of a release tagged (Θ, which is equal to r_t/R where r_t is the number of juvenile fish tagged and R is the total number of tagged and untagged fish in the release) is inversely related to the variance. Grouping constants of Equation 13 results in:

$$\mathrm{Var}(\hat{n}_1) = K_1 + K_2\left(\frac{1}{n_2}\right) \qquad [26]$$

where

$$K_1 = \frac{m_1(m_2-1)a_1(a_2-1)Nn_1(n_1-1)}{m_2(m_1-1)a_2(a_1-1)(N-1)} - n_1^2 \qquad [27]$$

[28]

$$K_2 = \frac{m_1 a_1 N n_1}{m_2 a_2 \Theta} - \frac{m_1(m_2-1)a_1(a_2-1)Nn_1(n_1-1)}{m_2(m_1-1)a_2(a_1-1)(N-1)}$$

and

[29]

$$Var(\hat{n}_1) = K_3 + K_4\left(\frac{1}{\Theta}\right)$$

where

[30]

$$K_3 = \frac{m_1(m_2-1)a_1(a_2-1)N(n_2-1)n_1(n_1-1)}{m_2(m_1-1)a_2(a_1-1)n_2(N-1)} - n_1^2$$

[31]

$$K_2 = \frac{m_1 a_1 N n_1}{m_2 a_2 n_2}$$

In a similar manner, by dividing Equation 26 through by the constant N, or by substituting r_t/R for Θ in Equation 29 and factoring out the constant R, it can also be shown that the variance is inversely related to the proportion of the catch sampled and to the number of fish tagged.

The variance is also a quadratic function of n_1:

[32]

$$Var(\hat{n}_1) = K_5 n_1^2 + K_6 n_1$$

where

[33]

$$K_5 = \frac{m_1(m_2-1)a_1(a_2-1)N(n_2-1)}{m_2(m_1-1)a_2(a_1-1)n_2(N-1)} - 1$$

[34]

$$K_6 = \frac{m_1a_1N}{m_2a_2n_2\Theta} - \frac{m_1(m_2-1)a_1(a_2-1)N(n_2-1)}{m_2(m_1-1)a_2(a_1-1)n_2(N-1)}$$

For strata with large catch and large sample sizes (large N and n_2), and with no significant loss of heads or tags, the value of constant K_5 approaches zero and the resulting variance is approximately proportional to n_1.

Often the objective of the CWT programs is based on relative precision rather than absolute precision. Absolute precision requires describing the desired precision in numbers of fish; relative precision requires the description be in percent of contribution. Therefore, relative precision is based on the coefficient of variation, defined as the square root of the variance divided by the number of fish of the release in the catch:

[35]

$$CV(\hat{n}_1) = \frac{\sqrt{Var(\hat{n}_1)}}{n_1} \times 100\%$$

where n_1 was previously shown to be equivalent to the expected value of the estimate. Substitution of the righthand side of Equation 32 into Equation 35 for the variance and assuming K_5 is approximately equal to zero results in:

[36]

$$CV(\hat{n}_1) = \frac{K_7\sqrt{n_1}}{n_1} = \frac{K_7}{\sqrt{n_1}}$$

where $K_7 = \sqrt{K_6}$ Therefore, although absolute precision is inversely related to n_1, relative precision is directly proportional to the square route of n_1 and will increase as the number of fish of a release increase in the catch.

Figures 7 - 10 depict the general relationship between the variance of contribution estimates and the rate of sampling, rate of tagging, and the catch composition. In order to decrease the variance, either 1). a larger proportion of the catch needs to be sampled; 2). a larger proportion of the release need to be tagged; 3). smaller numbers of fish of the release are caught by the fishery; 4). a combination of these three options. If a small proportion of a release is tagged and relatively high sampling rates are maintained, changes in the sampling rates will have little effect on the variance compared to the same change in tagging rates (Figure 7). This class of programs is 'tagging rate limited'. The sampling program is unable to reduce the variance below a ceratin value (even at 100% sampling rates) due to a the large amount of variance attributable to a small proportion of a release tagged. Conversely, in a CWT program in which a large proportion of a release was tagged and small proportions of the catches are sampled, an increase or decrease in rate of tagging will have relatively little effect on the variance compared to the same increase or decrease in rates of sampling (Figure 8). This class of CWT programs is 'sampling rate limited'. The tagging program is unable to reduce the variance below a certain value (even with a 100% tagging rate) due to the much larger amount of variance contributed to the program by small sampling rates. A good tagging program cannot compensate for a poor sampling program, and a good sampling program cannot compensate for a poor tagging program.

The effects of low tagging and/or sampling rates on the accuracy (as measured by the standard error) and relative accuracy (as measured by the coefficient of variation) are different for different numbers of a release in the catch. The standard error increases with larger numbers of a release in the catch and increases at a greater rate for releases which were tagged at low rates (Figure 9a) and catches which were sampled at low rates (Figure 10a). However, there is relatively more effect of low tagging and sampling rates on the relative accuracy at low numbers of fish of a release in the catch (Figures 9b and 10b).

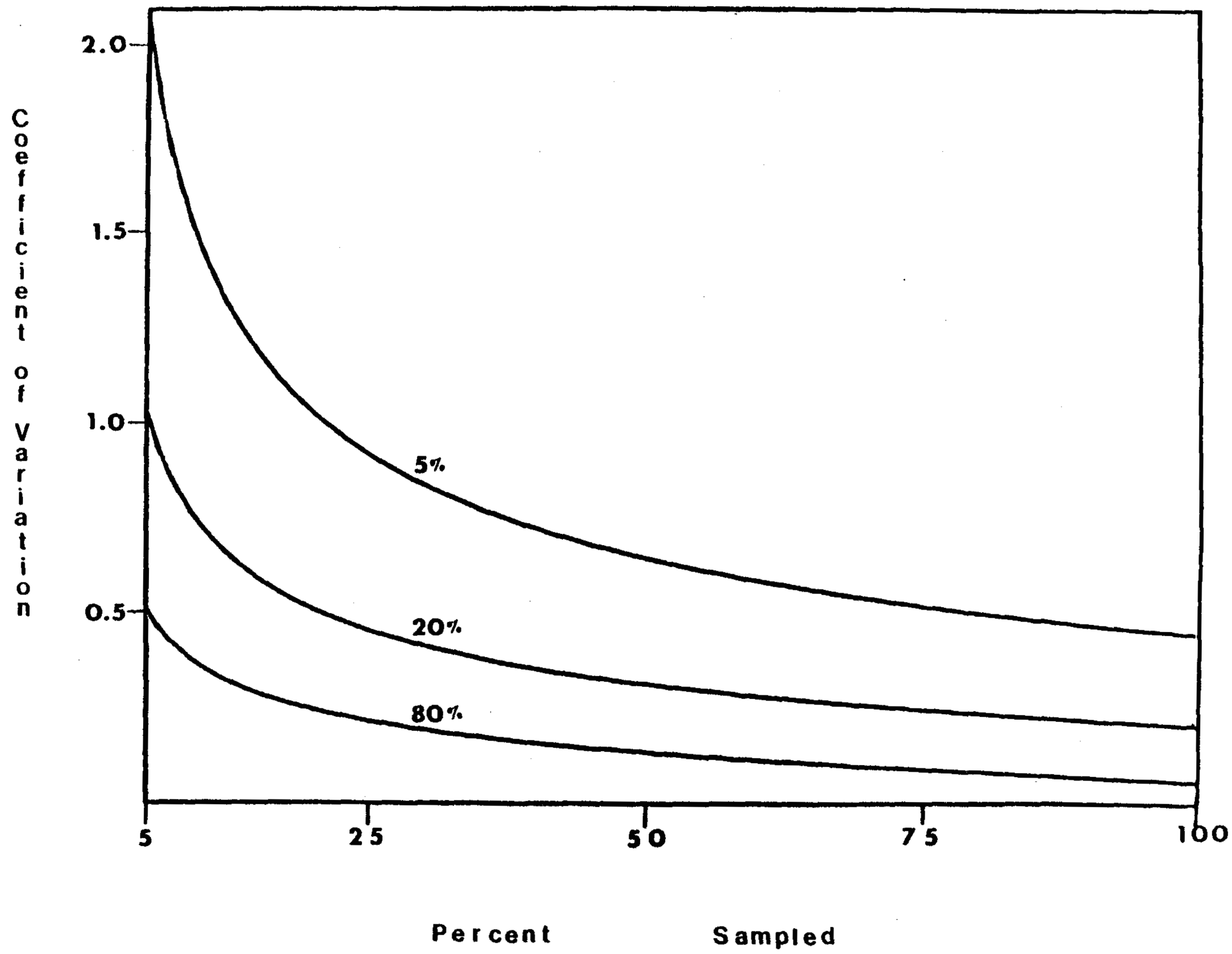


Figure 7. Change in the coefficient of variation of the estimated number of a release in the catch ($\hat{n}$) with increasing or decreasing percentages of catch sampled. The example assumes that no tags were lost, 5% of the heads were lost, 100 fish of the release were caught, and the percent of the release tagged was either 5%, 20%, or 80%.

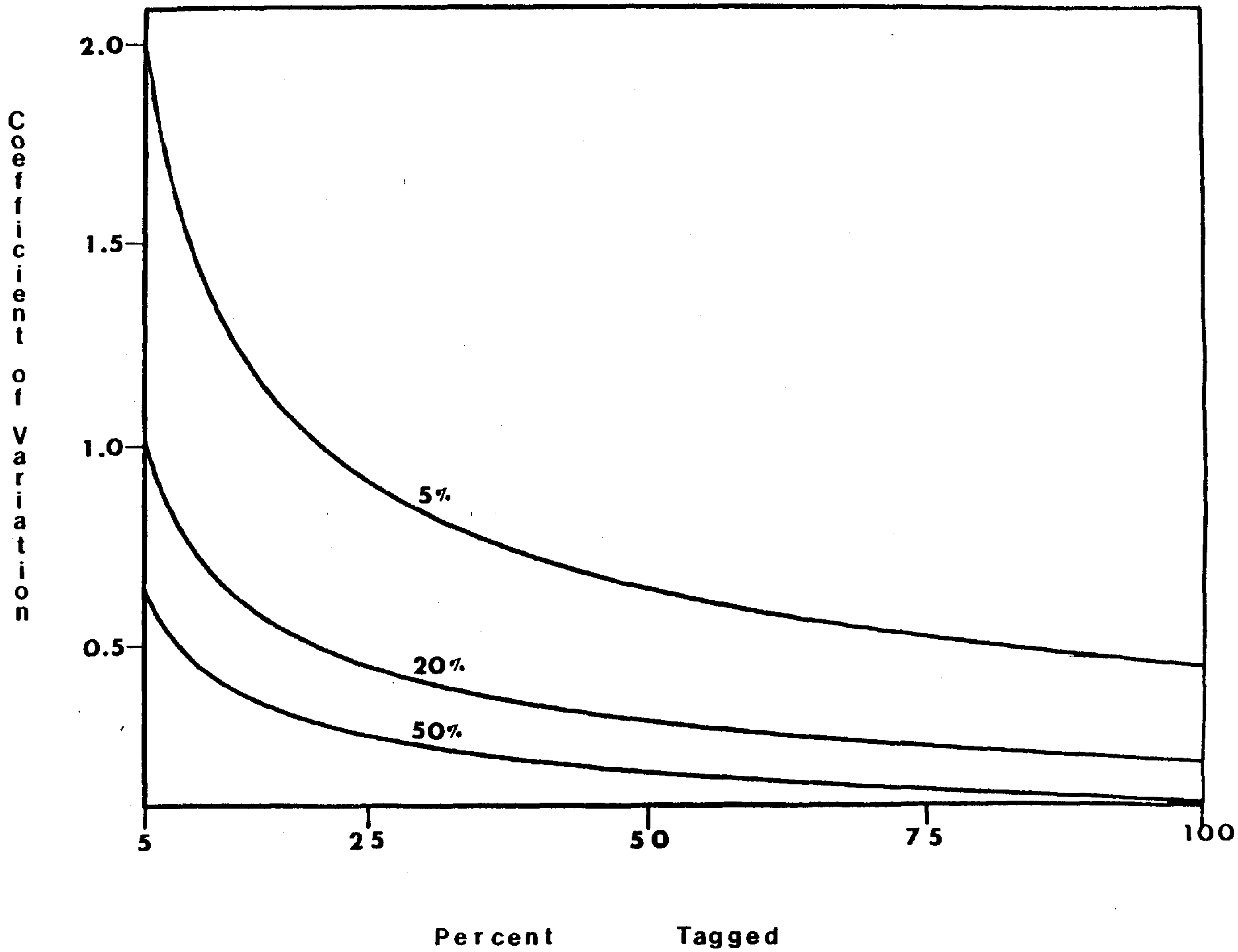
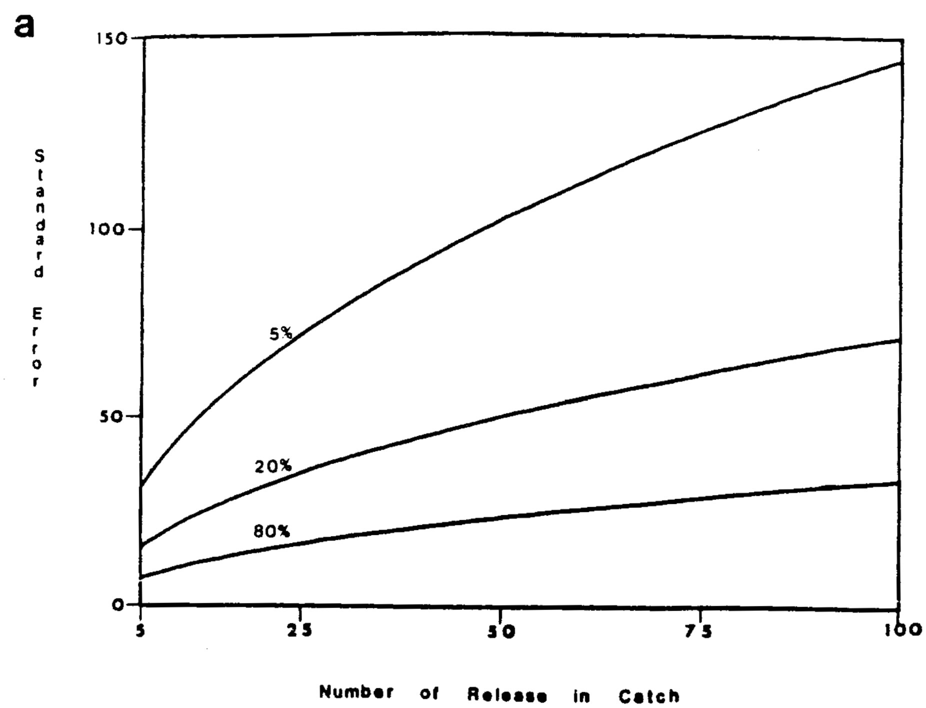


Figure 8. Change in the coefficient of variation of the estimated number of a release in the catch ($\hat{n}$) with increasing or decreasing percentages of a release tagged. The example assumes that no tags were lost, 5% of the heads were lost, 100 fish of the release were caught, and the percent of the catch sampled was either 5%, 20%, or 50%.



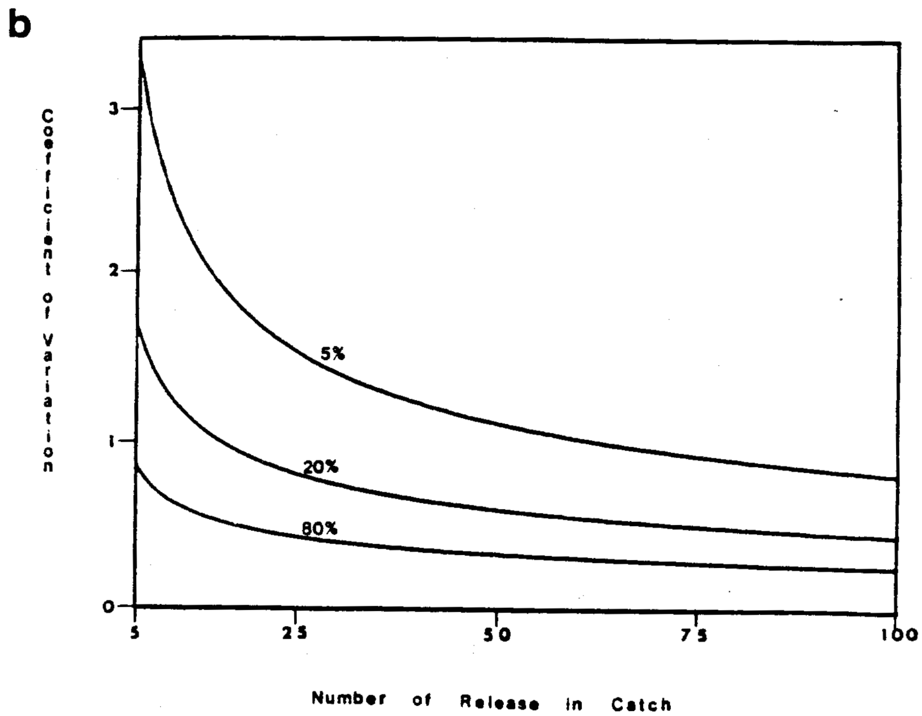


Figure 9. Change in the a) standard error; and b) coefficient of variation of the estimated number of a release in the catch ($\hat{n}$) with increasing or decreasing actual number of fish of the release in the catch. The example assumes that no tags were lost, 5% of the heads were lost, 20% of the catch was sampled, and the percent of the release tagged was either 5%, 20%, or 80%.

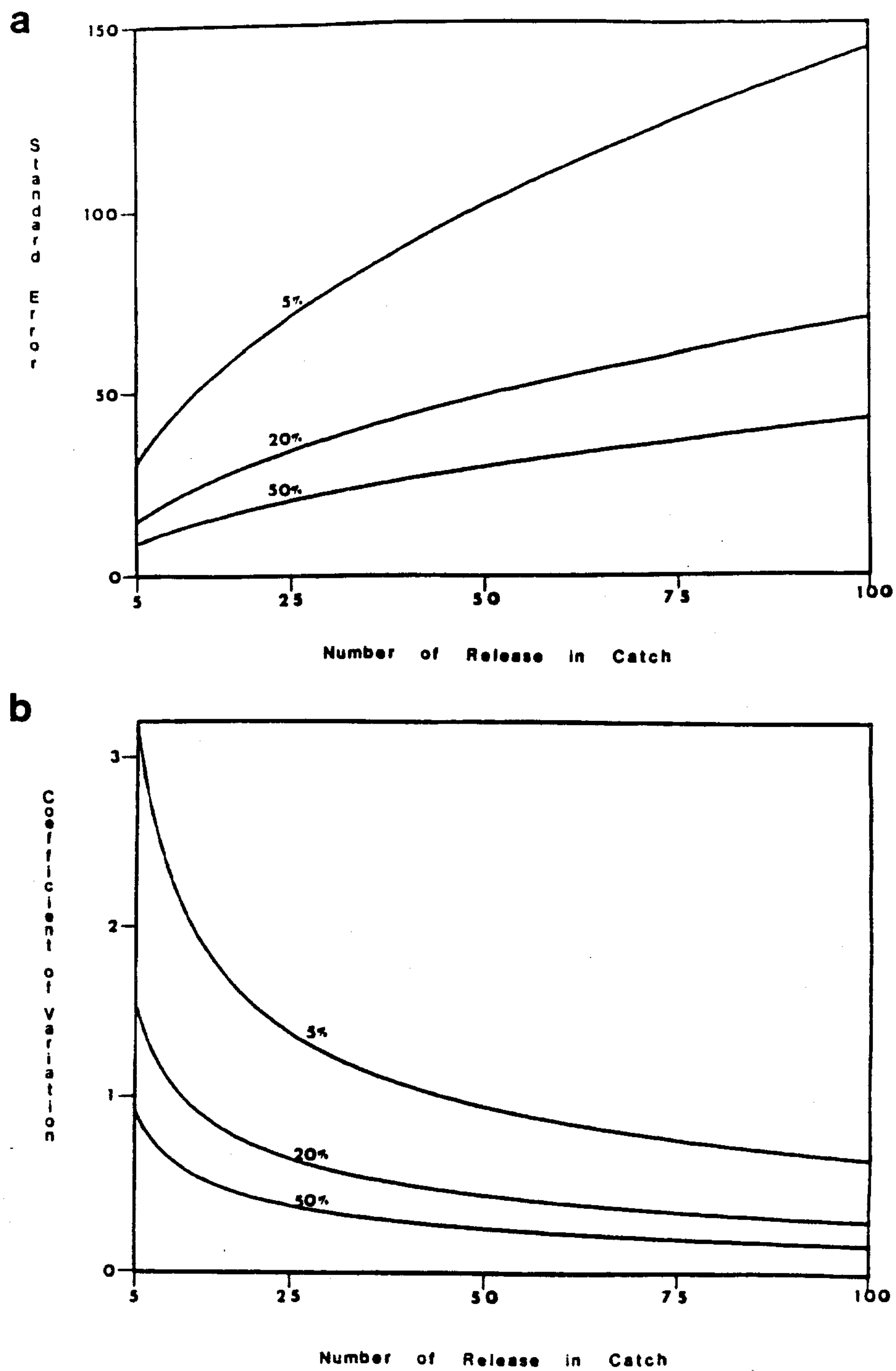


Figure 10. Change in the a) standard error; and b) coefficient of variation of the estimated number of a release in the catch ($\hat{n}$) with increasing or decreasing actual number of fish of the release in the catch. The example assumes that no tags were lost, 5% of the heads were lost, 20% of the release was tagged, and the percent of the catch sampled was either 5%, 20%, or 50%.

OPTIMUM ALLOCATION OF RESOURCES

The planning of a CWT project includes activities and a budget. Smolts must be tagged, commercial catches must be inspected for missing adipose fins, marked heads must be transported and dissected, tags need to be decoded, data analyzed, and results reported. The amount of money put into the project will obviously affect the precision of the estimates that come out. The more money involved, the better the chance of getting more precise estimates. As discussed previously, minimization of the variance is also dependent upon allocation of financial resources, as evidence by levels of tagging and sampling activities, to different programs. Of the project activities, the tagging of smolts and the checking of the commercial catch are generally the most expensive. So once the funding level for the project is decided, what is the most effective way to distribute money between these two activities?

Without dealing in specifics, several general rules on the amount and distribution of funds needed to minimize the variance of the estimates can be derived from the variance formula. First, the variance can be simplified. En Equation 13, if the catch (N) and the sample size (n_2) are large numbers, the equation

$$\left(\frac{N}{N-1}\right)\left(\frac{n_2}{n_2-1}\right) \cong 1 \quad [37]$$

is approximately true. Therefore, Equation 13 can be reduced to:

$$\mathrm{Var}(\hat{n}_1) = \frac{m_1(m_2-1)a_1(a_2-1)n_1(n_1-1)}{m_2(m_1-1)a_2(a_1-1)} + \frac{m_1a_1Nn_1}{m_2a_2n_2\Theta} - n_1^2 \quad [38]$$

Because this step produces a simpler equation of variance with little change in the relative influence of numbers tagged and numbers sampled, Equation 38 is a better subject for investigating the optimal allocation of funds to CWT projects.

The allocation of monies between tagging and sampling activities is investigated first. The combined cost of sampling and tagging in each project can be expressed as the sum of products:

[39]

$$C_T = C_{r_t} r_t + C_{n_2} n_2$$

[40]

$$C_T = C_{r_t} r_t + \sum_{i=1}^{s} C_{n_{2}i} n_{2i}$$

[41]

$$C_T = \sum_{j=1}^{t} C_{r_{t}j} r_{tj} + \sum_{i=1}^{s} C_{n_{2}i} n_{2i}$$

where $C_{n_{2}i}$ and $C_{r_{t}j}$ are the cost of sampling one adult fish for a missing adipose fin and for tagging one smolt, respectively, r_t is the number of smolts tagged, C_T is the total dollar cost of the combined activities, and i and j designate the particular catch stratum and release respectively. Equation 39 describes the situation of one release and one stratum, Equation 40 describes the situation of one release and many catch strata, and Equation 41 describes the situation of many releases and many catch strata. All three equations are linear cost functions with fixed per unit costs $C_{n_{2}i}$ and $C_{r_{t}j}$. Usually unit costs are a function of the units produced with cost per unit dropping as initial fixed costs in a project are defrayed (such as purchase of tagging machines and dissecting microscopes). However, variable costs, such as labor, travel, and commodities would have cost functions similar to those of Equations 39 - 41.

The variance can be minimized with respect to r_t and n_2 by using Lagrange's method with the cost functions as constraints (Appendix F). The principal result is that the variance will be minimal when half the money is spent on tagging and half on sampling. This conclusion is true when there is but one tagged release and one sampling stratum, when there are several sampling strata and one release, and when there are multiple releases and multiple sampling strata. Size of catches, fraction of the catch sampled (as long as n_2 is large), size of the contribution of the release to the catch and distribution of fish across catch strata, fraction of the release tagged (as long as this fraction is not close to 1.0), rate of head loss, and rate of tag loss have no effect on this conclusion. Note that this 50/50 rule refers to variable costs only. Money spent on capital improvements and equipment will affect the cost functions by reducing the per unit costs C_{n_2} and C_{r_t} and thereby will reduce variances.

It is a fact of CWT programs that tagging precedes sampling, usually by several years. If extra money is available for the program after tagging has been completed, this money can be allocated to the sampling program, with a corresponding improvement in the variance. However, this improvement is not as great as it would have been if one-half of the extra money had been spent on the tagging program. If less money is available for the sampling program than previously projected, the resulting variance is larger than anticipated and would have been smaller if monies from the tagging program would have been carried over to the sampling program.

Although the 50/50 rule is robust, its use is predicated upon funds being optimally spent once they have been allocated to tagging or sampling programs. Obviously funds spent on a poorly designed catch sampling program would have been better spent on an optimally designed tagging program and funds spent on a poorly designed tagging program would have been better spent on a well designed catch sampling program.

For situations with multiple catch sampling strata and/or several releases, optimal allocation of funds among releases can be estimated by continuing with the minimization procedure outlined above. From Appendix G, the ratio of funding for catch sampling strata i and h that will produce a minimum variance is:

[42]

$$\frac{F_{n_2 i}}{F_{n_2 h}} = \left[\frac{m_{2h} a_{2h} m_{1i} a_{1i} N_i C_{n_2 i} \sum_{j=1}^{t} \frac{n_{1ij}}{\Theta_j}}{m_{2i} a_{2i} m_{1h} a_{1h} N_h C_{n_2 h} \sum_{j=1}^{t} \frac{n_{1hj}}{\Theta_j}} \right]^{\frac{1}{2}}$$

For the special case when there are multiple catch sampling strata but only one release of interest, Equation 42 reduces to:

[43]

$$\frac{F_{n_2 i}}{F_{n_2 h}} = \left[\frac{m_{2h} a_{2h} m_{1i} a_{1i} N_i C_{n_2 i} n_{1ij}}{m_{2i} a_{2i} m_{1h} a_{1h} N_h C_{n_2 h} n_{1hj}} \right]^{\frac{1}{2}}$$

Note that there are a series of ratios in Equation 43:

$$\left(\frac{m_{2h}}{m_{1h}}\right), \left(\frac{a_{2h}}{a_{1h}}\right), \left(\frac{m_{2i}}{m_{1i}}\right)^{-1}, \left(\frac{a_{2i}}{a_{1i}}\right)^{-1}, \left(\frac{N_i}{N_h}\right), \left(\frac{n_{1i}}{n_{1h}}\right), \text{ and } \left(\frac{C_{n_2 i}}{C_{n_2 h}}\right)$$

These ratios correspond to rates of head retention and of tags successfully decoded within stratum i and in stratum h and catches, contributions, and unit costs in these two strata. Inspection of these ratios in the context of Equations 42 and 43 shows that to minimize variances, more money should be spent in strata with a greater rate of head loss, poorer rate of successfully decoding tags, greater catches, greater contributions, and greater per unit costs. If there is no difference in rates of head retention, rates of decoded tags, catches, contributions, or unit costs among the strata, monies are best allocated equally among strata. Again, these rules apply to variable costs. When feasible, fixed costs should be encumbered to improve variances when rates of head retention, of successfully decoding tags and lower unit costs can be improved with capital expenditures.

Optimal allocation of monies among releases in a tagging program are found with this same procedure (Appendix G). The ratio of funding for tagging of release j and release 1 that will produce a minimal variance is

$$\frac{F_{r_t j}}{F_{r_t 1}} = \left(\frac{C_{r_t j} R_j \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i}}}{C_{r_t 1} R_1 \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1i1}}{m_{2i} a_{2i} n_{2i}}} \right)^{\frac{1}{2}} \tag{44}$$

If only one stratum is present in the catch sampling program or if rates of head and tag loss and catch sampling rates are the same across all strata, Equation 44 reduces to:

$$\frac{F_{r_t j}}{F_{r_t 1}} = \left(\frac{C_{r_t j} R_j n_{1ij}}{C_{r_t 1} R_1 n_{1i1}} \right)^{\frac{1}{2}} \tag{45}$$

Again note that there are a series of ratios in Equation 45:

$$\left(\frac{C_{r_t j}}{C_{r_t 1}} \right), \left(\frac{R_j}{R_1} \right), \text{ and } \left(\frac{n_{1ij}}{n_{1i1}} \right)$$

These ratios correspond to the unit cost of tagging releases, to the size of the release, and to the size of the ultimate contribution of the release to the catch. Inspection of these ratios in the context of Equations 44 and 45 shows that to minimize variances, more money should be spent to tag more of larger releases, more of releases with greater contributions, and more of releases with poor per unit costs. Note that $n_l = SR$ where S is the survival rate from time of release to time of capture. If this identity is placed into Equation 45 the result is:

$$\frac{F_{r_t j}}{F_{r_t 1}} = \frac{R_j}{R_1}\left(\frac{C_{r_t j} S_j}{C_{r_t 1} S_1}\right)^{\frac{1}{2}} \qquad [46]$$

Equation 46 predicts that minimal variance will result when more money is spent on tagging more of the releases with better survival rates.

Specific calculations concerning percision and funding can be made if a priori estimates of unit costs, catches, contributions, and rates of head loss and of undecoded tags are available. If funding of variable costs is specified and not precision, the 50/50 rule, Equations 42 and 44, and Equation 13 can be used to optimally allocate these funds between and within programs and estimate the resulting variance before the money is spent. However, if a degree of precision is the objective, these same equations along with the cost functions (Equations 39 - 41) can be used to estimate the money needed to cover the variable, and, in this case, the fixed costs of attaining the objectives.

DISCUSSION

Statistical methodology describing the Southeastern Alaska tagging and sampling program is developed. Results obtained from the variance and covariance equations compare well with those obtained by empirical means. A robust 50/50 rule can be applied when decisions concerning allocation of resources between tagging and sampling programs are made. Ratios of rates of losing heads and losing tags, catch, size of release, unit cost of tagging, unit cost of sampling, and number of release fish in catch help determine optimum allocation of sampling effort across strata and of tagging effort across releases. Other concerns not addressed in the present study are the consequences of variability in estimates of the commercial catch or proportion of a release tagged, the magnitude of nonsampling errors in the tagging and sampling programs and their effect on contribution and variance estimates, and estimation of the contribution of untagged releases to the commercial fisheries. However, the methodology developed in the present study can serve as a statistical foundation from which to investigate other problems. For example, by considering the proportion of a release tagged and the catch as random variables, not constants, and assigning probability distributions to these variables, maximum likelihood theory can used to derive estimates of contribution rates and the associated variances and covariances.

The advantage of describing a tagging and sampling program in terms of a probability distribution is obvious. Equations which calculate the statistical parameters of interest can be proven to be mathematically correct and exact. Disagreements in methodology are confined to the capability of tagging and sampling programs to meet the assumptions implicit in the statistical model. Failure of empirical data to confirm the correctness of the model does not indicate a failure of the model itself, but may be symptomatic of deviations from the generally assumed randomness of tagging and sampling programs. Such failures have been noted by Webb (1985) and de Libero (1986) for West Coast coded wire tagging and sampling programs. With appropriate modifications, the basic statistical methodology for estimating the contributions and associated variances and covariances of tagged releases will be applicable to other coastal fisheries.

It should be emphasized that documentation of the contribution of a release to the commercial fisheries is, at present, possible only if fish which are representive of each hatchery release are tagged. Increasing production from Southeastern Alaska hatchery facilities coupled with decreasing fiscal resources will encourage the tendency to release fish which are not represented by a coded wire tagged group. If this occurs, future documentation of contribution rates will predictably result in questionable and often disputed estimates calculated by generally unproven and ambiguous estimation techniques.

As the number of hatchery produced salmon increase in Southeastern Alaskan waters, the need for timely and precise management of commercial fisheries likewise increases. The tendency to reduce coded wire tagging programs while increasing production levels will result in a decrease in the accuracy and precision of both inseason estimates and postseason analysis. Uncertainties concerning the levels of exploitation to which natural stocks are being subjected and the pattern of entry of hatchery stocks into the commercial fisheries will limit the ability of fishery managers to optimally regulate the harvest. A consequence of a lack of accurate estimates of the number of hatchery and natural stock salmon being harvested will be an underutilization of hatchery produced fish by a conservatively managed fishery or an overexploitation of natural stocks in a highly exploited fishery. The severe consequences of adversely affecting natural stocks, monetary loss to commercial fishermen, and loss knowledge concerning the biological attributes of tagged releases should be carefully considered when decreases in tagging programs are proposed.

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APPENDICES

APPENDIX A

Derivation of the formula for the expected value of m_c will follow the methods presended by Freund and Walpole (1980) to determine formulas for the means of the simple binomial and hypergeometric distributions (pp. 168 - 181). The mean (or expected value) of the number of coded wire tags (CWTs) of a given tag code expected to be recovered in a defined stratum is the sum of the products of all values of m_c and the probability of recovering and decoding that number of tags. This is expressed by Equation 8 in text. Replacing x_3 by m_2 (since $p(m_c) = 0$ for all $x_3 > m_2$ and for all $m_c > x_3$):

[A1]

$$E[m_c] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} [1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] \sum_{m_c=0}^{m_2} m_c \left[\frac{\binom{x_3}{m_c}\binom{m_1-x_3}{m_2-m_c}}{\binom{m_1}{m_2}} \right]$$

Since the term corresponding to $m_c = 0$ can be omitted, and canceling the m_c against the first factor of $m_c!$, Equation A2 is obtained:

[A2]

$$E[m_c] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} [1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] \sum_{m_c=1}^{m_2} \left[\frac{x_3!}{[m_c-1]!\,[x_3-m_c]!} \right] \frac{\binom{m_1-x_3}{m_2-m_c}}{\binom{m_1}{m_2}}$$

Factoring out $x_3\binom{m_1}{m_2}^{-1}$

[A3]

$$E[m_c] = \sum_{x_1=0}^{n_1}\ \sum_{x_2=0}^{x_1}\ \sum_{x_3=0}^{x_2} \binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}}\right]$$

$$\left[\frac{\binom{x_2}{x_3}\ \binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}}\right]\ x_3\binom{m_1}{m_2}^{-1}\sum_{m_c=1}^{m_2}\binom{x_3-1}{m_c-1}\binom{m_1-x_3}{m_2-m_c}$$

and since

[A4]

$$\sum_{m_c=1}^{m_2}\binom{x_3-1}{m_c-1}\binom{m_1-x_3}{m_2-m_c} = \sum_{(m_c-1)=0}^{m_2-1}\binom{x_3-1}{m_c-1}\binom{m_1-x_3}{[m_2-1]-[m_c-1]} =$$

$$\binom{m_1-1}{m_2-1}$$

by substitution, Equation A3 reduces to:

[A5]

$$E[m_c] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} [1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] x_3 \binom{m_1}{m_2}^{-1} \binom{m_1-1}{m_2-1}$$

and multiplying the two combinations $\binom{m_1}{m_2}^{-1} \binom{m_1-1}{m_2-1}$

[A6]

$$E[m_c] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} [1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] x_3 \left[\frac{m_2}{m_1} \right]$$

Removing all constants from the summation $\sum_{x_3=0}^{x_2}$ and replacing x_2 by a_2:

[A7]

$$E[m_c] = \left[\frac{m_2}{m_1}\right] \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \binom{n_1}{x_1} \Theta^{x_1} [1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}}\right]$$

$$\sum_{x_3=0}^{a_2} x_3 \frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}}$$

Equation A7 is simplified by evaluating the sum of the products of x_3 and the hypergeometric distribution over all possible values of x_3 by the same proceedures outlines in Equations A1 to A6. Omiting the term corresponding to $x_3 = 0$, canceling out x_3 with the first factor of $x_3!$, factoring out $x_2\binom{a_1}{a_2}^{-1}$ and since:

[A8]

$$\sum_{x_3=1}^{a_2} \binom{x_2-1}{x_3-1}\binom{a_1-x_2}{a_2-x_3} = \binom{a_1-1}{a_2-1}$$

by substitution, Equation A7 is equal to:

[A9]

$$E[m_c]=\left[\frac{m_2}{m_1}\right] \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}}\right]\left[\frac{a_2}{a_1}\right]x_2$$

In a manner similar to the preceeding proceedures, by removing all constants from the summation $\sum_{x_2=0}^{x_1}$ and replacing x_2 by a_2, we get:

[A10]

$$E[m_c]=\left[\frac{m_2}{m_1}\right] \left[\frac{a_2}{a_1}\right] \sum_{x_1=0}^{n_1} \binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1} \sum_{x_2=0}^{x_1} x_2\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}}$$

by the methods outlined in previous equations, Equation A10 simplifies to:

[A11]

$$E[m_c]=\left[\frac{m_2}{m_1}\right] \left[\frac{a_2}{a_1}\right] \left[\frac{n_2}{N}\right] \sum_{x_1=0}^{n_1} x_1 \binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1}$$

APPENDIX B

The equations which compute the variance of m_c and the variance of n_1, and which estimate the variance of $\hat{n}_1$ will be derived following the methods presented in Freund and Walpole (1980) and in Appendix A. Since

[B1]

$$Var(m_c) = E[m_c(m_c-1)] + E[m_c] - (E[m_c])^2$$

each of the three expected values on the right hand side of the equality will be evaluated and combined to produce an equation for the variance. The expected value $E[m_c(m_c-1)]$ will be evaluated first. The expected value of $m_c(m_c-1)$ is the sum of the products of this quantity and the probabilities of it being realized:

[B2]

$$E[m_c(m_c-1)] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1}\theta^{x_1}[1-\theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] \sum_{m_c=0}^{m_2} m_c[m_c-1] \frac{\binom{x_3}{m_c}\binom{m_1-x_3}{m_2-m_c}}{\binom{m_1}{m_2}}$$

If terms corresponding to $m_c = 0$ and $m_c = 1$ are omitted (since at these values the terms in the summation equal 0) and the m_c and $m_c - 1$ terms are canceled against the first two factors of $m_c!$, Equation B3 results:

[B3]

$$E[m_c(m_c-1)] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} [1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] \sum_{m_c=2}^{m_2} \frac{x_3!}{[m_c-2]!\,[x_3-m_c]!} \frac{\binom{m_1-x_3}{m_2-m_c}}{\binom{m_1}{m_2}}$$

Factoring out $x_3[x_3-1]\binom{m_1}{m_2}^{-1}$

[B4]

$$E[m_c(m_c-1)] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] \quad x_3[x_3-1]\binom{m_1}{m_2}^{-1} \sum_{m_c=2}^{m_2} \binom{x_3-2}{m_c-2}\binom{m_1-x_3}{m_2-m_c}$$

Since

[B5]

$$\sum_{m_c=2}^{m_2} \binom{x_3-2}{m_c-2}\binom{m_1-x_3}{m_2-m_c} = \sum_{(m_c-2)=0}^{m_2-2} \binom{x_3-2}{m_c-2}\binom{m_1-x_3}{[m_2-2]-[m_c-2]} =$$

$$\binom{m_1-2}{m_2-2}$$

and substituting back into the Equation B4:

[B6]

$$E[m_c(m_c-1)] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} [1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3} \binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] x_3[x_3-1] \binom{m_1}{m_2}^{-1} \binom{m_1-2}{m_2-2}$$

and multiplying the two combinations $\binom{m_1}{m_2}^{-1} \binom{m_1-2}{m_2-2}$ results in Equation [B7].

[B7]

$$E[m_c(m_c-1)] = \sum_{x_1=0}^{n_1} \sum_{x_2=0}^{x_1} \sum_{x_3=0}^{x_2} \binom{n_1}{x_1} \Theta^{x_1} [1-\Theta]^{n_1-x_1} \left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}} \right]$$

$$\left[\frac{\binom{x_2}{x_3} \binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}} \right] x_3[x_3-1] \left[\frac{m_2(m_2-1)}{m_1(m_1-1)} \right]$$

Removing all constants from the summation $\sum_{x_3=0}^{x_2}$

and replacing x_2 with a_2

[B8]

$$E[m_c(m_c-1)] = \left[\frac{m_2(m_2-1)}{m_1(m_1-1)}\right]\sum_{x_1=0}^{n_1}\sum_{x_2=0}^{x_1}\binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1}\left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}}\right]$$

$$\sum_{x_3=0}^{a_2} x_3[x_3-1]\frac{\binom{x_2}{x_3}\binom{a_1-x_2}{a_2-x_3}}{\binom{a_1}{a_2}}$$

In a similar manner, Equation B8 can be evaluated by canceling the x_3 and x_3-1 terms, factoring out

$x_2[x_2-1]\binom{a_1}{a_2}^{-1}$

and summing the remaining products of combinations results in

[B9]

$$E[m_c(m_c-1)] = \left[\frac{m_2(m_2-1)}{m_1(m_1-1)}\right]\sum_{x_1=0}^{n_1}\sum_{x_2=0}^{x_1}\binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1}\left[\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}}\right]$$

$$x_2[x_2-1]\quad\left[\frac{a_2(a_2-1)}{a_1(a_1-1)}\right]$$

Rearranging Equation B9 and replacing x_1 with n_2 produces:

[B10]

$$E[m_c(m_c-1)] = \left[\frac{m_2(m_2-1)}{m_1(m_1-1)}\right]\left[\frac{a_2(a_2-1)}{a_1(a_1-1)}\right]\sum_{x_1=0}^{n_1} \binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1}$$

$$\sum_{x_2=0}^{n_2} x_2[x_2-1]\frac{\binom{x_1}{x_2}\binom{N-x_1}{n_2-x_2}}{\binom{N}{n_2}}$$

Using the same mathematical steps outlined in the preceeding Equations B2 to B7 and again in Equations B8 to B9, Equation B11 cam be simplified to:

[B11]

$$E[m_c(m_c-1)] = \left[\frac{m_2(m_2-1)}{m_1(m_1-1)}\right]\left[\frac{a_2(a_2-1)}{a_1(a_1-1)}\right]\sum_{x_1=0}^{n_1} \binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1}$$

$$x_1[x_1-1]\left[\frac{n_2(n_2-1)}{N(N-1)}\right]$$

and rearranged:

[B12]

$$E[m_c(m_c-1)] =$$

$$\left[\frac{m_2(m_2-1)}{m_1(m_1-1)}\right]\left[\frac{a_2(a_2-1)}{a_1(a_1-1)}\right]\left[\frac{n_2(n_2-1)}{N(N-1)}\right]\sum_{x_1=0}^{n_1} x_1[x_1-1] \binom{n_1}{x_1}\Theta^{x_1}[1-\Theta]^{n_1-x_1}$$

Removing the $x_1 = 0$ and $x_1 = 1$ term, factoring out n_1, n_1-1, and Θ^2 results in:

[B13]

$$E[m_c(m_c-1)] = \left[\frac{m_2(m_2-1)}{m_1(m_1-1)}\right]\left[\frac{a_2(a_2-1)}{a_1(a_1-1)}\right]\left[\frac{n_2(n_2-1)}{N(N-1)}\right]$$

$$n_1[n_1-1]\ \Theta^2\sum_{x_1=2}^{n_1} x_1[x_1-1] \binom{n_1-2}{x_1-2}\Theta^{x_1-2}[1-\Theta]^{[(n_1-2)-(x_1-2)]}$$

and since

[B14]

$$\sum_{x_1-2=0}^{n_1-2} x_1[x_1-1] \binom{n_1-2}{x_1-2}\Theta^{x_1-2}[1-\Theta]^{[(n_1-2)-(x_1-2)]} = 1$$

Substitution results in

[B15]

$$E[m_c(m_c-1)] = \left[\frac{m_2(m_2-1)}{m_1(m_1-1)}\right]\left[\frac{a_2(a_2-1)}{a_1(a_1-1)}\right]\left[\frac{n_2(n_2-1)}{N(N-1)}\right] n_1[n_1-1]\ \Theta^2$$

Using the results of Appendix A, it is known that

[B16]

$$E[m_c] = \left[\frac{m_2}{m_1}\right]\ \left[\frac{a_2}{a_1}\right]\ \left[\frac{n_2}{N}\right]\ \Theta n_1$$

and likewise,

[B17]

$$(E[m_c])^2 = \left[\frac{m_2 a_2 n_2 \Theta n_1}{m_1 a_1 N}\right]^2$$

Substitution of Equations B15, B16, and B17 into Equation B1 results in an equation for the variance of m_c:

[B18]

$$\mathrm{Var}(m_c) = \left(\frac{m_2 a_2 n_2 n_1 \Theta^2 (m_2-1)(a_2-1)(n_2-1)(n_1-1)}{m_1 a_1 N (m_1-1)(a_1-1)(N-1)}\right)$$

$$+ \frac{m_2 a_2 n_2 n_1 \Theta}{m_1 a_1 N} - \left(\frac{m_2 a_2 n_2 n_1 \Theta}{m_1 a_1 N}\right)^2$$

Since

[B19]

$$\hat{n}_1 = \left(\frac{m_1}{m_2}\right)\left(\frac{a_1}{a_2}\right)\left(\frac{N}{n_2}\right)\left(\frac{m_c}{\Theta}\right)$$

then

[B20]

$$\mathrm{Var}(\hat{n}_1) = \left(\frac{m_1}{m_2}\right)^2\left(\frac{a_1}{a_2}\right)^2\left(\frac{N}{n_2}\right)^2\left(\frac{1}{\Theta}\right)^2\mathrm{Var}(m_c)$$

Substituting for $\mathrm{Var}(m_c)$ and simplifying:

[B21]

$$\mathrm{Var}(\hat{n}_1) = \left(\frac{m_1a_1Nn_1(m_2-1)(a_2-1)(n_2-1)(n_1-1)}{m_2a_2n_2(m_1-1)(a_1-1)(N-1)}\right) + \frac{m_1a_1Nn_1}{m_2a_2n_2\Theta} - n_1^2$$

An estimate of $\mathrm{Var}(\hat{n}_1)$, denoted as $S^2(\hat{n}_1)$ can be obtained by substituting the estimated value of n_1, $\hat{n}_1$, into Equation B21 for n_1:

[B22]

$$S^2(\hat{n}_1) = \left[m_c\left(\frac{Nm_1a_1}{m_2a_2n_2\Theta}\right)^2\right]\left[1 - m_c + \left(\frac{(m_2-1)(a_2-1)(n_2-1)}{(m_1-1)(a_1-1)(N-1)}\right)\left(\frac{m_1a_1Nm_c}{m_2a_2n_2} - \Theta\right)\right]$$

APPENDIX C

The estimate,

[C1]

$$\left(\frac{m_2}{m_2-1}\right)\left(\frac{m_1-1}{m_1}\right)\left(\frac{a_2}{a_2-1}\right)\left(\frac{a_1-1}{a_1}\right)\left(\frac{n_2}{n_2-1}\right)\left(\frac{N-1}{N}\right) S^2(\hat{n}_1)$$

is an unbiased estimator of $Var(\hat{n}_1)$ if its expected value is equal to $Var(\hat{n}_1)$:

[C2]

$$E\left[\left(\frac{m_2}{m_2-1}\right)\left(\frac{m_1-1}{m_1}\right)\left(\frac{a_2}{a_2-1}\right)\left(\frac{a_1-1}{a_1}\right)\left(\frac{n_2}{n_2-1}\right)\left(\frac{N-1}{N}\right) S^2(\hat{n}_1)\right] =$$

$$\frac{m_1a_1Nn_1(m_2-1)(a_2-1)(n_2-1)(n_1-1)}{m_2a_2n_2(m_1-1)(a_1-1)(N-1)} + \frac{m_1a_1Nn_1}{m_2a_2n_2\Theta} - n_1^2$$

To simplify the notation, constants will be grouped. Let

[C3]

$$C_1 = \frac{m_2a_2n_2}{m_1a_1N}$$

and

$$C_2 = \frac{(m_2-1)(a_2-1)(n_2-1)}{(m_1-1)(a_1-1)(N-1)} \tag{C4}$$

Substitute these 2 constants and the expression equal to $S^2(\hat{n}_1)$ into expression C1 and evaluate the expected value will result in:

$$E\left[\ C_1C_2^{-1}\left(\frac{C_1^{-2}m_c}{\Theta^2}\right)\left(1 - m_c + C_2C_1^{-1}m_c - \Theta C_2\right)\ \right] \tag{C5}$$

Removing constants from the expectation operator and rearranging results in

$$\left(\frac{1-C_2\Theta}{C_1C_2\Theta^2}\right)E[m_c] + \left(\frac{C_2-C_1}{C_1^2C_2\Theta^2}\right)E[m_c^2] \tag{C6}$$

Given that

$$E[m_c] = C_1\Theta n_1 \tag{C7}$$

(Equation A12 with C_1 replacing the constants) and

$$E[m_c^2] = C_1C_2\Theta n_1(n_1-1) + C_1\Theta n_1 \tag{C8}$$

(Equations B15 and B16 with C_1 and C_2 replacing the constants) substitution of these quantities in Expression C6 will yield:

[C9]

$$\left(\frac{1-C_2\Theta}{C_1C_2\Theta^2}\right)C_1\Theta n_1 + \left(\frac{C_2-C_1}{C_1^2C_2\Theta^2}\right)(C_1C_2\Theta n_1(n_1-1) + C_1\Theta n_1)$$

Combining, canceling, and rearranging constants:

[C10]

$$\frac{C_2n_1(n_1-1)}{C_1} + \frac{n_1}{C_1\Theta} - n_1^2$$

and replacing the constants C_1 and C_2 by the combinations of constants they represent results in an expression equal to the variance of n_1 (Equation C2)

[C11]

$$\frac{m_1a_1Nn_1(m_2-1)(a_2-1)(n_2-1)(n_1-1)}{m_2a_2n_2(m_1-1)(a_1-1)(N-1)} + \frac{m_1a_1Nn_1}{m_2a_2n_2\Theta} - n_1^2$$

Therefore, $S^2(n_1)$ is an estimate of the variance of the estimated number of release fish in a catch stratum which is biased by the inverse of the correction term:

$$\left(\frac{m_1}{m_1-1}\right)\left(\frac{m_2-1}{m_2}\right)\left(\frac{a_1}{a_1-1}\right)\left(\frac{a_2-1}{a_2}\right)\left(\frac{N}{N-1}\right)\left(\frac{n_2-1}{n_2}\right)$$

Appendix D.

In order to derive an equation which estimates the covariance between m_{c_A} and m_{c_B}, the following relationships will be used. Given the general multivariate hypergeometric distribution:

[D1]

$$p(y_{1_A}, y_{1_B}; n, N, k_{1_A}, k_{1_B}) = \frac{\binom{k_{1_A}}{y_{1_A}} \binom{k_{1_B}}{y_{1_B}} \binom{N-k_{1_A}-k_{1_B}}{n-y_{1_A}-y_{1_B}}}{\binom{N}{n}}$$

where y_{1_A} and y_{1_B} are random variables, and N, n, k_{1_A}, and k_{1_B} are constants defining the distribution, the covariance between y_{1_A} and y_{1_B} is:

[D2]

$$\mathrm{Cov}(y_{1_A}; y_{1_B}) = -\frac{nk_{1_A}k_{1_B}}{N^2}\left[\frac{N-n}{N-1}\right]$$

(see Johnson and Kotz, 1969). The covariance can also be defined as:

[D3]

$$\mathrm{Cov}(y_{1_A}; y_{1_B}) = E(y_{1_A}y_{1_B}) - E(y_{1_A})E(y_{1_B})$$

The quantity $E(y_{1_A} y_{1_B})$ can also be defined as the sum of all products of $y_{1_A} y_{1_B}$ and the probability of realizing such a product.

[D4]

$$E(y_{1_A} y_{1_B}) = \sum_{y_{1_A}}^{n} \sum_{y_{1_B}}^{n} y_{1_A} y_{1_B} \frac{\binom{k_{1_A}}{y_{1_A}} \binom{k_{1_B}}{y_{1_B}} \binom{N-k_{1_A}-k_{1_B}}{n-y_{1_A}-y_{1_B}}}{\binom{N}{n}}$$

To evaluate $E(y_{1_A} y_{1_B})$ of Equation D4 in terms of the constants N, n, k_{1_A}, and k_{1_B}, Equations D2 and D3 are used. Since

[D5]

$$E(y_{1_A}) = \frac{k_{1_A} n}{N}$$

and

[D6]

$$E(y_{1_B}) = \frac{k_{1_B} n}{N}$$

$E(y_{1_A} y_{1_B})$ can be solved for.

[D7]

$$E(y_{1_A} y_{1_B}) = E(y_{1_A})E(y_{1_B}) + \mathrm{Cov}(y_{1_A} ; y_{1_B})$$

[D8]

$$E(y_{1_A} y_{1_B}) = \frac{k_{1_A} k_{1_B} n^2}{N^2} - \frac{n k_{1_A} k_{1_B}}{N^2} \left[\frac{N-n}{N-1}\right]$$

[D9]

$$E(y_{1_A} y_{1_B}) = \frac{k_{1_A} k_{1_B} n}{N^2} \left[n - \frac{N-n}{N-1}\right]$$

The following generalized equality will be used to derive an equation for the covariance between m_{c_A} and m_{c_B}:

[D10]

$$\sum_{y_{1_A}}^{n} \sum_{y_{1_B}}^{n} y_{1_A} y_{1_B} \frac{\binom{k_{1_A}}{y_{1_A}} \binom{k_{1_B}}{y_{1_B}} \binom{N-k_{1_A}-k_{1_B}}{n-y_{1_A}-y_{1_B}}}{\binom{N}{n}} = \frac{k_{1_A} k_{1_B} n}{N^2} \left[n - \frac{N-n}{N-1}\right]$$

To estimate $Cov(m_{c_A};m_{c_B})$, the sum of all possible products of $m_{c_A}m_{c_B}$ and the probability of realizing this quantity (Equation 21 in the text) will be evaluated:

[D11]

$$E(m_{c_A}m_{c_B}) = \sum_{x_{1_A}=0}^{n_{1_A}} \sum_{x_{1_B}=0}^{n_{1_B}} \sum_{x_{2_A}=0}^{x_{1_A}} \sum_{x_{2_B}=0}^{x_{1_B}} \sum_{x_{3_A}=0}^{x_{2_A}} \sum_{x_{3_B}=0}^{x_{2_B}} \binom{n_{1_A}}{x_{1_A}} \Theta_A^{x_{1_A}}[1-\Theta_A]^{n_{1_A}-x_{1_A}}$$

$$\binom{n_{1_B}}{x_{1_B}} \Theta_B^{x_{1_B}}[1-\Theta_B]^{n_{1_B}-x_{1_B}} \frac{\binom{x_{1_A}}{x_{2_A}}\binom{x_{1_B}}{x_{2_B}}\binom{N-x_{1_A}-x_{1_B}}{n_2-x_{2_A}-x_{2_B}}}{\binom{N}{n_2}}$$

$$\frac{\binom{x_{2_A}}{x_{3_A}}\binom{x_{2_B}}{x_{3_B}}\binom{a_1-x_{2_A}-x_{2_B}}{a_2-x_{3_A}-x_{3_B}}}{\binom{a_1}{a_2}} \sum_{m_{c_A}=0}^{m_2} \sum_{m_{c_B}=0}^{m_2} m_{c_A}m_{c_B} \frac{\binom{x_{3_A}}{m_{c_A}}\binom{x_{3_B}}{m_{c_B}}\binom{m_1-x_{3_A}-x_{3_B}}{m_2-m_{c_A}-m_{c_B}}}{\binom{m_1}{m_2}}$$

By employing the generalized relationship in Equation D10, Equation D11 simplifies to:

[D12]

$$E(m_{c_A} m_{c_B}) = \sum_{x_{1_A}=0}^{n_{1_A}} \sum_{x_{1_B}=0}^{n_{1_B}} \sum_{x_{2_A}=0}^{x_{1_A}} \sum_{x_{2_B}=0}^{x_{1_B}} \sum_{x_{3_A}=0}^{x_{2_A}} \sum_{x_{3_B}=0}^{x_{2_B}} \binom{n_{1_A}}{x_{1_A}} \Theta_A^{x_{1_A}} [1-\Theta_A]^{n_{1_A}-x_{1_A}}$$

$$\binom{n_{1_B}}{x_{1_B}} \Theta_B^{x_{1_B}} [1-\Theta_B]^{n_{1_B}-x_{1_B}} \frac{\binom{x_{1_A}}{x_{2_A}} \binom{x_{1_B}}{x_{2_B}} \binom{N-x_{1_A}-x_{1_B}}{n_2-x_{2_A}-x_{2_B}}}{\binom{N}{n_2}}$$

$$\frac{\binom{x_{2_A}}{x_{3_A}} \binom{x_{2_B}}{x_{3_B}} \binom{a_1-x_{2_A}-x_{2_B}}{a_2-x_{3_A}-x_{3_B}}}{\binom{a_1}{a_2}} \quad x_{3_A} x_{3_B} \frac{m_2}{m_1^2} \left[m_2 - \frac{m_1-m_2}{m_1-1} \right]$$

Removing constants from the summations over all possible values of x_{3_A} and x_{3_B} and replacing the x_{2_A} and x_{2_B} of the summation by a_2 results in:

[D13]

$$E(m_{c_A} m_{c_B}) = \frac{m_2}{m_1^2}\left[m_2 - \frac{m_1 - m_2}{m_1 - 1}\right] \sum_{x_{1_A}=0}^{n_{1_A}} \sum_{x_{1_B}=0}^{n_{1_B}} \sum_{x_{2_A}=0}^{x_{1_A}} \sum_{x_{2_B}=0}^{x_{1_B}} \binom{n_{1_A}}{x_{1_A}} \Theta_A^{x_{1_A}} [1-\Theta_A]^{n_{1_A}-x_{1_A}}$$

$$\binom{n_{1_B}}{x_{1_B}} \Theta_B^{x_{1_B}} [1-\Theta_B]^{n_{1_B}-x_{1_B}} \frac{\binom{x_{1_A}}{x_{2_A}} \binom{x_{1_B}}{x_{2_B}} \binom{N - x_{1_A} - x_{1_B}}{n_2 - x_{2_A} - x_{2_B}}}{\binom{N}{n_2}}$$

$$\sum_{x_{3_A}=0}^{a_2} \sum_{x_{3_B}=0}^{a_2} x_{3_A} x_{3_B} \frac{\binom{x_{2_A}}{x_{3_A}} \binom{x_{2_B}}{x_{3_B}} \binom{a_1 - x_{2_A} - x_{2_B}}{a_2 - x_{3_A} - x_{3_B}}}{\binom{a_1}{a_2}}$$

The summation of x_{3_A} and x_{3_B} over all values of a_1 and a_2 is in a form similar to Equation D10. By repeating the derivation process outlined from Equations D11 to D13 for simplification of the summation of x_{2_A} and x_{2_B} over all possible values, Equation D13 will simplify to

[D14]

$$E(m_{c_A} m_{c_B}) = \left[\left[\frac{m_2 a_2 n_2}{m_1^2 a_1^2 N^2} \right] \left[m_2 - \frac{m_1 - m_2}{m_1 - 1} \right] \left[a_2 - \frac{a_1 - a_2}{a_1 - 1} \right] \left[n_2 - \frac{N - n_2}{N - 1} \right] \right]$$

$$\sum_{x_{1_A}=0}^{n_{1_A}} \sum_{x_{1_B}=0}^{n_{1_B}} x_{1_A} x_{1_B} \binom{n_{1_A}}{x_{1_A}} \Theta_A^{x_{1_A}} [1-\Theta_A]^{n_{1_A}-x_{1_A}} \binom{n_{1_B}}{x_{1_B}} \Theta_B^{x_{1_B}} [1-\Theta_B]^{n_{1_B}-x_{1_B}}$$

which can be rearranged as

[D15]

$$E(m_{c_A} m_{c_B}) = \left[\left[\frac{m_2 a_2 n_2}{m_1^2 a_1^2 N^2} \right] \left[m_2 - \frac{m_1 - m_2}{m_1 - 1} \right] \left[a_2 - \frac{a_1 - a_2}{a_1 - 1} \right] \left[n_2 - \frac{N - n_2}{N-1} \right] \right]$$

$$\sum_{x_{1_A}=0}^{n_{1_A}} x_{1_A} \binom{n_{1_A}}{x_{1_A}} \Theta_A^{x_{1_A}} [1-\Theta_A]^{n_{1_A}-x_{1_A}} \sum_{x_{1_B}=0}^{n_{1_B}} x_{1_B} \binom{n_{1_B}}{x_{1_B}} \Theta_B^{x_{1_B}} [1-\Theta_B]^{n_{1_B}-x_{1_B}}$$

Since the summations in Equation 15 are simply the expected values of x_{1_A} and x_{1_B} for binomially distributed variables (see Appendix A), the summations will reduce to:

[D16]

$$\sum_{x_{1_A}=0}^{n_{1_A}} x_{1_A} \binom{n_{1_A}}{x_{1_A}} \Theta_A^{x_{1_A}} [1-\Theta_A]^{n_{1_A}-x_{1_A}} = \Theta_A n_{1_A}$$

and

[D17]

$$\sum_{x_{1_B}=0}^{n_{1_B}} x_{1_B} \binom{n_{1_B}}{x_{1_B}} \Theta_B^{\,x_{1_B}} [1-\Theta_B]^{\,n_{1_B}-x_{1_B}} = \Theta_B n_{1_B}$$

Therefore,

[D18]

$$E(m_{c_A} m_{c_B}) = \Theta_A \Theta_B n_{1_A} n_{1_B} \left[\frac{m_2 a_2 n_2}{m_1^2 a_1^2 N^2} \right] \left[m_2 - \frac{m_1 - m_2}{m_1 - 1} \right]$$

$$\left[a_2 - \frac{a_1 - a_2}{a_1 - 1} \right] \left[n_2 - \frac{N - n_2}{N - 1} \right]$$

As was demonstrated in Appendix A

[D19]

$$E(m_{c_A}) = \left(\frac{m_2}{m_1} \right) \left(\frac{a_2}{a_1} \right) \left(\frac{n_2}{N} \right) \Theta_A n_{1_A}$$

and

[D20]

$$E(m_{c_B}) = \left(\frac{m_2}{m_1} \right) \left(\frac{a_2}{a_1} \right) \left(\frac{n_2}{N} \right) \Theta_B n_{1_B}$$

By substitution defined by Equations D18, D19, and D20 into Equation D3, an equation which quantifies the covariance of m_{c_A} and m_{c_B} is derived:

[D21]

$$\mathrm{Cov}(m_{c_A};m_{c_B}) =$$

$$\Theta_A\Theta_B n_{1_A} n_{1_B}\left(\frac{m_2 a_2 n_2}{m_1{}^2 a_1{}^2 N^2}\right)\left(m_2 - \frac{m_1-m_2}{m_1-1}\right)\left(a_2 - \frac{a_1-a_2}{a_1-1}\right)\left(n_2 - \frac{N-n_2}{N-1}\right)$$

$$- \left(\frac{m_2{}^2 a_2{}^2 n_2{}^2}{m_1{}^2 a_1{}^2 N^2}\right)\Theta_A\Theta_B n_{1_A} n_{1_B}$$

Using the general rule

[D22]

$$\mathrm{Cov}(k_1 y_1;k_2 y_2) = k_1 k_2 \mathrm{Cov}(y_1;y_2)$$

where k_1 and k_2 are constants and y_1 and y_2 are random variables, and substituting the estimated values of n_{1_A} and n_{1_B} into Equation D21, an estimate of the covariance between n_{1_A} and n_{1_B} is

[D23]

$$\mathrm{C\hat{o}v}(n_{1_A};n_{1_B}) = \frac{m_{c_A} m_{c_B} m_1{}^2 a_1{}^2 N^2}{\Theta_A\Theta_B m_2{}^3 a_2{}^3 n_2{}^3}\left[\left(m_2 - \frac{(m_1-m_2)}{m_1-1}\right)\right.$$

$$\left.\left(a_2 - \frac{(a_1-a_2)}{a_1-1}\right)\left(n_2 - \frac{(N-n_2)}{N-1}\right) - m_2 a_2 n_2\right]$$

By substituting $\hat{n}_{1_A}$ and $\hat{n}_{1_B}$ into Equation D23, it can be simplified:

[D24]

$$C\hat{o}v(n_{1_A};n_{1_B}) = \hat{n}_{1_A}\hat{n}_{1_B}\left[\frac{m_1(m_2-1)a_1(a_2-1)N(n_2-1)}{m_2(m_1-1)a_2(a_1-1)n_2(N-1)} - 1\right]$$

Appendix E

Appendix E Table 1. Estimated contribution of chinook salmon release represented by tag codes B40907 and B40908 to the commercial troll fishery and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1983. Of the total release of 135,163 chinook salmon, 94,723 (or 70.08%) possessed tag code B40907 and 37,737 (or 27.92%) possessed tag code B40908. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	Quadrant	Stat. Week	Number of Tags Recovered B40907	Number of Tags Recovered B40908	Estimated Contribution B40907	Estimated Contribution B40908	Estimated Contribution Combined	Variance of Estimated Contribution B40907	Variance of Estimated Contribution B40908	Covariance	Variance of Difference MCBH	Variance of Difference Empirical	Coefficient of Variance MCBH	Coefficient of Variance Empirical
Troll	N.W.	23	10	2	93	46	79	759	1031	-2	1795	2123	0.534	0.580
Troll	N.W.	24	30	10	121	102	116	368	929	-1	1299	393	0.311	0.171
Troll	N.W.	27	9	3	48	40	45	203	487	-1	691	60	0.580	0.171
Troll	N.W.	28	10	3	44	33	41	147	329	-1	477	117	0.537	0.266
Troll	N.W.	29	8	2	36	23	32	128	236	-0	364	182	0.589	0.416
Troll	N.W.	30	6	1	181	76	151	5234	5669	-28	10960	11128	0.692	0.697
Troll	N.W.	35	3	1	21	18	20	125	289	-0	414	12	1.019	0.174
Troll	S.W.	1 -16	1	0	4	0	3	14	0	0	14	19	1.210	1.409
Troll	S.W.	23	6	3	55	69	59	446	1521	-10	1986	199	0.752	0.238
Troll	S.W.	24	18	8	44	49	45	61	246	-1	308	25	0.390	0.111
Troll	S.W.	27	3	2	21	36	25	130	599	-1	731	206	1.064	0.565
Troll	S.W.	28	6	1	35	15	29	171	203	-0	374	420	0.658	0.697
Troll	S.W.	29	4	0	15	0	10	38	0	0	38	211	0.593	1.399
Troll	N.E.	1 -16	1	1	3	6	4	4	36	0	40	15	1.710	1.047
Troll	N.E.	23	11	6	25	34	28	32	160	-0	192	85	0.502	0.334
Troll	N.E.	24	15	6	35	35	35	46	169	-0	215	0	0.420	0.000
Troll	N.E.	27	10	2	39	20	34	113	173	-0	287	380	0.505	0.581
Troll	N.E.	28	13	4	55	43	51	177	409	-1	588	157	0.471	0.243
Troll	N.E.	29	13	4	38	29	35	71	181	-0	252	73	0.452	0.243
Troll	N.E.	30	5	0	13	0	10	23	0	0	22	180	0.489	1.399
Troll	N.E.	35	2	0	18	0	13	146	0	0	146	330	0.930	1.398
Troll	S.E.	1 -16	5	0	14	0	10	25	0	0	25	196	0.500	1.399
Troll	S.E.	23	11	5	57	65	59	230	772	-12	1025	65	0.539	0.136
Troll	S.E.	24	26	9	106	92	102	321	849	-6	1183	194	0.336	0.136
Troll	S.E.	27	48	17	132	117	127	212	675	-15	918	214	0.238	0.115
Troll	S.E.	28	18	5	59	41	54	129	293	-3	427	316	0.385	0.331
Troll	S.E.	29	12	2	36	15	30	71	99	-1	171	440	0.435	0.697
Troll	S.E.	30	6	4	10	16	11	6	48	-0	55	42	0.646	0.565
Troll	S.E.	35	2	0	12	0	8	54	0	0	54	134	0.889	1.400
Average											864	618	0.634	0.583
Total					1370	1020	1269	9481	15401					
95% Confidence Intervals					+/-191	+/-243								

Appendix E Table 2. Estimated contribution of chinook salmon release represented by tag codes B40907 and B40908 to the commercial net and trap fisheries and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1983. Of the total release of 135,163 chinook salmon, 94,723 (or 70.08%) possessed tag code B40907 and 37,737 (or 27.92%) possessed tag code B40908. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	District	Stat. Week	Number of Tags Recovered B40907	Number of Tags Recovered B40908	Estimated Contribution B40907	Estimated Contribution B40908	Estimated Contribution Combined	Variance of Estimated Contribution B40907	Variance of Estimated Contribution B40908	Covariance	Variance of Difference MCBH	Variance of Difference Empirical	Coefficient of Variance MCBH	Coefficient of Variance Empirical
Seine	101	29	3	1	10	8	9	21	60	-2	84	3	0.966	0.183
Seine	101	30	2	1	6	8	7	13	54	-1	70	3	1.229	0.254
Seine	101	31	8	4	22	27	23	34	153	-3	194	30	0.600	0.236
Seine	101	32	7	1	16	6	13	18	25	-0	44	99	0.523	0.785
Seine	104	28	1	0	5	0	3	18	0	0	18	23	1.245	1.407
Seine	104	31	1	0	7	0	5	40	0	0	40	47	1.296	1.405
Seine	104	32	0	1	0	13	4	0	155	0	155	169	3.366	3.515
Seine	109	31	0	1	0	12	3	0	127	0	127	140	3.337	3.504
Seine	110	30	1	0	1	0	1	1	0	0	1	2	0.980	1.386
Seine	110	31	1	0	2	0	1	1	0	0	1	2	0.910	1.287
Seine	112	27	2	1	3	4	3	1	10	0	11	1	1.063	0.321
Seine	112	29	2	2	3	8	5	2	25	0	27	24	1.126	1.062
Seine	112	30	1	0	2	0	2	4	0	0	3	6	0.993	1.404
Seine	113	30	1	0	4	0	3	14	0	0	14	19	1.212	1.412
Gillnet	101	25	1	0	5	0	4	20	0	0	20	25	1.240	1.386
Gillnet	101	26	4	3	14	26	17	33	195	-1	229	146	0.882	0.705
Gillnet	101	28	1	0	1	0	1	1	0	0	1	2	0.980	1.386
Gillnet	101	29	1	1	2	5	3	2	20	-0	23	9	1.650	1.032
Gillnet	101	31	2	0	6	0	4	11	0	0	11	34	0.798	1.403
Gillnet	101	35	1	0	30	0	21	792	0	0	792	885	1.323	1.399
Gillnet	106	25	1	0	8	0	6	58	0	0	58	68	1.294	1.401
Gillnet	106	26	5	1	19	10	16	52	82	-1	136	91	0.711	0.581
Gillnet	106	27	0	1	0	10	3	0	84	0	84	101	3.201	3.510
Gillnet	106	28	4	1	10	6	9	14	33	-0	48	14	0.779	0.421
Gillnet	106	29	2	0	5	0	4	9	0	0	9	30	0.770	1.406
Gillnet	111	25	1	0	4	0	3	13	0	0	13	17	1.225	1.401
Gillnet	111	27	2	0	4	0	3	4	0	0	4	16	0.707	1.413
Gillnet	115	26	0	1	0	7	2	0	43	0	43	50	3.250	3.505
Gillnet	115	33	1	0	11	0	8	92	0	0	92	110	1.277	1.396
Fishtrap	101	28	4	0	16	0	11	43	0	0	43	244	0.587	1.399
Fishtrap	101	29	1	0	4	0	3	10	0	0	10	14	1.169	1.384
Fishtrap	101	30	1	0	3	0	2	5	0	0	5	8	1.096	1.386
Fishtrap	101	31	0	1	0	7	2	0	39	0	39	47	3.193	3.505
Fishtrap	101	32	1	1	1	4	2	1	9	0	10	5	1.550	1.096
Average											72	73	1.368	1.449
Total					224	161	205	1325	1114					
95% Confidence Intervals					+/-71	+/-65								

Appendix E Table 3. Estimated contribution of chinook salmon release represented by tag codes 042430 and 042431 to the commercial net and trap fisheries and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1984. Of the total release of 73,540 chinook salmon, 10,335 (or 14.05%) possessed tag code 042430 and 10,792 (or 14.68%) possessed tag code 042431. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	District	Stat. Week	Number of Tags Recovered 042430	Number of Tags Recovered 042431	Estimated Contribution 042430	Estimated Contribution 042431	Estimated Contribution Combined	Variance of Estimated Contribution 042430	Variance of Estimated Contribution 042431	Covariance	Variance of Difference MCBH	Variance of Difference Empirical	Coefficient of Variance MCBH	Coefficient of Variance Empirical
Seine	110	31	1	0	8	0	4	51	0	0	51	59	1.905	2.049
Seine	113	30	1	0	22	0	11	441	0	0	441	464	1.994	2.045
Seine	110	30	0	1	0	7	3	0	40	0	40	46	1.817	1.948
Seine	112	28	0	1	0	7	4	0	40	0	40	47	1.800	1.951
Seine	112	29	0	2	0	15	8	0	103	0	103	237	1.289	1.956
Gillnet	101	30	0	1	0	16	8	0	245	0	245	262	1.892	1.956
Gillnet	106	31	0	1	0	43	22	0	1755	0	1755	1863	1.900	1.958
Average											382	425	1.800	1.980
Total					30	88	59	492	2183					
95% Confidence Intervals					+/-43	+/-92								

Appendix E Table 4. Estimated contribution of coho salmon release represented by tag codes 042240 and 042256 to the commercial troll fishery and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1984. Of the total release of 44,500 coho salmon, 10,020 (or 22.52%) possessed tag code 042240 and 10,454 (or 23.49%) possessed tag code 042256. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	Quadrant	Stat. Week	Number of Tags Recovered 042240	Number of Tags Recovered 042256	Estimated Contribution 042240	Estimated Contribution 042256	Estimated Contribution Combined	Variance of Estimated Contribution 042240	Variance of Estimated Contribution 042256	Covariance	Variance of Difference MCBH	Variance of Difference Empirical	Coefficient of Variance MCBH	Coefficient of Variance Empirical
Troll	N.W.	27	0	2	0	578	283	0	166339	0	166339	334091	1.442	2.043
Troll	N.W.	28	2	3	51	80	65	1243	2033	-1	3277	826	0.881	0.443
Troll	N.W.	29	7	7	174	181	177	4126	4499	-2	8630	56	0.524	0.042
Troll	N.W.	30	39	27	725	524	627	12752	9635	-9	22405	40572	0.239	0.321
Troll	N.W.	35	6	13	118	268	191	2215	5234	-1	7452	22251	0.451	0.780
Troll	S.W.	28	4	3	64	50	57	957	784	-1	1743	194	0.731	0.244
Troll	S.W.	29	3	8	48	132	89	704	2045	-3	2754	7169	0.590	0.952
Troll	S.W.	30	4	4	68	71	69	1087	1186	-1	2275	9	0.687	0.043
Troll	S.W.	35	6	6	118	124	121	2216	2418	-3	4640	26	0.563	0.042
Troll	N.E.	27	1	0	27	0	14	685	0	0	685	712	1.921	1.958
Troll	N.E.	28	0	1	0	13	6	0	146	0	146	159	1.960	2.045
Troll	N.E.	29	0	1	0	16	8	0	238	0	238	254	1.978	2.043
Troll	N.E.	30	1	1	18	19	18	305	333	-0	639	1	1.377	0.054
Troll	S.E.	27	1	1	12	12	12	121	133	-0	254	0	1.353	0.000
Troll	S.E.	28	0	1	0	15	7	0	210	0	210	227	1.967	2.045
Troll	S.E.	29	2	0	18	0	9	149	0	0	149	336	1.304	1.959
Troll	S.E.	30	5	4	58	49	54	619	541	-1	1161	93	0.637	0.180
Troll	S.E.	35	4	3	97	76	87	2266	1855	-8	4138	449	0.739	0.243
Average											12619	22635	1.075	0.858
Total					1596	2208	1895	29445	197629					
95% Confidence Intervals					+/-336	+/-871								

Appendix E Table 5. Estimated contribution of coho salmon release represented by tag codes 042240 and 042256 to the commercial net and trap fisheries and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1984. Of the total release of 44,500 coho salmon, 10,020 (or 22.52%) possessed tag code 042240 and 10,454 (or 23.49%) possessed tag code 042256. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	Quadrant	Stat. Week	Number of Tags Recovered 042240	Number of Tags Recovered 042256	Estimated Contribution 042240	Estimated Contribution 042256	Estimated Contribution Combined	Variance of Estimated Contribution 042240	Variance of Estimated Contribution 042256	Covariance	Variance of Difference MCBH	Variance of Difference Empirical	Coefficient of Variance MCBH	Coefficient of Variance Empirical
Seine	104	28	0	1	0	23	11	0	510	0	510	540	1.985	2.043
Seine	104	29	1	0	19	0	10	340	0	0	340	360	1.903	1.958
Seine	104	30	0	1	0	29	14	0	791	0	791	820	2.007	2.044
Seine	104	32	2	1	55	29	42	1446	789	-1	2238	688	1.126	0.625
Seine	104	33	2	3	33	52	42	519	849	-0	1368	352	0.872	0.442
Seine	104	34	0	1	0	51	25	0	2576	0	2576	2639	2.019	2.043
Seine	104	35	2	1	41	21	31	801	437	-1	1240	386	1.119	0.625
Seine	101	31	1	0	9	0	5	76	0	0	76	85	1.854	1.960
Seine	101	32	1	1	26	27	27	659	718	-10	1396	1	1.389	0.037
Seine	101	33	2	1	45	24	35	961	527	-5	1497	465	1.121	0.625
Seine	102	33	1	1	47	49	48	2102	2290	-38	4467	4	1.401	0.042
Seine	101	34	12	18	227	355	289	4038	6598	-20	10677	16385	0.357	0.443
Seine	102	34	1	0	396	0	202	154545	0	0	154545	156719	1.945	1.958
Seine	101	35	15	31	198	426	310	2393	5367	-32	7823	52243	0.286	0.738
Seine	102	35	0	2	0	66	32	0	2094	0	2094	4409	1.408	2.043
Seine	106	35	1	4	17	73	44	282	1201	-10	1503	3041	0.874	1.243
Seine	101	36	9	8	106	99	103	1144	1112	-9	2274	60	0.464	0.075
Seine	102	36	10	8	50	42	46	198	175	-1	375	68	0.422	0.180
Seine	101	37	0	5	0	141	69	0	3758	0	3758	19771	0.891	2.043
Seine	101	38	1	2	44	93	68	1887	4020	-88	6082	2329	1.146	0.709
Seine	101	40	0	1	0	12	6	0	126	0	126	138	1.955	2.046

Appendix E Table 5. Estimated contribution of coho salmon release represented by tag codes 042240 and 042256 to the commercial net and trap fisheries and comparison of the associated variance (cont.). Fish were released from the Whitman Lake hatchery in 1984. Of the total release of 44,500 coho salmon, 10,020 (or 22.52%) possessed tag code 042240 and 10,454 (or 23.49%) possessed tag code 042256. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	Quadrant	Stat. Week	Number of Tags Recovered 042240	Number of Tags Recovered 042256	Estimated Contribution 042240	Estimated Contribution 042256	Estimated Contribution Combined	Variance of Estimated Contribution 042240	Variance of Estimated Contribution 042256	Covariance	Variance of Difference MCBH	Variance of Difference Empirical	Coefficient of Variance MCBH	Coefficient of Variance Empirical
Gillnet	101	27	1	0	12	0	6	121	0	0	121	136	1.848	1.959
Gillnet	101	30	0	1	0	11	6	0	117	0	117	128	1.951	2.041
Gillnet	106	31	1	1	26	28	27	673	734	-1	1408	1	1.389	0.037
Gillnet	101	32	2	0	19	0	10	164	0	0	164	366	1.311	1.959
Gillnet	101	33	4	4	53	55	54	632	691	-5	1333	5	0.681	0.042
Gillnet	106	33	2	1	17	9	13	133	73	0	206	69	1.080	0.625
Gillnet	101	34	5	4	71	59	65	940	822	-1	1764	138	0.642	0.180
Gillnet	106	34	2	0	22	0	11	213	0	0	213	469	1.320	1.958
Gillnet	101	35	6	9	59	93	76	524	858	-3	1388	1121	0.493	0.443
Gillnet	106	35	4	2	39	20	30	345	189	-0	535	352	0.770	0.624
Gillnet	101	36	13	8	96	61	79	607	410	-1	1019	1173	0.404	0.434
Gillnet	106	36	0	2	0	26	13	0	316	0	316	686	1.387	2.044
Gillnet	108	36	1	0	5	0	3	21	0	0	21	26	1.758	1.956
Gillnet	101	37	11	6	97	55	77	761	455	-2	1220	1759	0.455	0.546
Gillnet	106	37	2	2	22	23	23	226	247	-1	474	1	0.956	0.044
Gillnet	101	38	19	14	143	110	126	924	746	-2	1675	1086	0.324	0.261
Fishtrap	101	32	1	1	4	4	4	14	15	0	29	0	1.239	0.000
Fishtrap	101	33	1	0	4	0	2	14	0	0	14	18	1.721	1.952
Fishtrap	101	34	1	0	6	0	3	34	0	0	34	40	1.804	1.956
Fishtrap	101	35	4	3	23	18	21	112	92	-1	206	26	0.688	0.244
Fishtrap	101	36	3	3	14	14	14	49	54	-0	102	0	0.723	0.000
Average											5193	6407	1.178	1.029
Total					2045	2198	2121	177898	39757					
95% Confidence Intervals					+/-827	+/-391								

Appendix E Table 6. Estimated contribution of coho salmon release represented by tag codes 042257 and 042258 to the commercial troll fishery and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1984. Of the total release of 328,559 coho salmon, 10,349 (or 3.15%) possessed tag code 042257 and 10,159 (or 3.09%) possessed tag code 042258. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	Quadrant	Stat. Week	Number of Tags Recovered 042257	Number of Tags Recovered 042258	Estimated Contribution 042257	Estimated Contribution 042258	Estimated Contribution Combined	Variance of Estimated Contribution 042257	Variance of Estimated Contribution 042258	Covariance	Variance of Difference MCBH	Variance of Difference Empirical	Coefficient of Variance MCBH	Coefficient of Variance Empirical
Troll	N.W.	27	3	1	6198	2105	4170	12783750	4425587	-5027	17219390	16755180	0.995	0.982
Troll	N.W.	28	7	4	1328	773	1053	250374	148556	-159	399247	308011	0.600	0.527
Troll	N.W.	29	15	7	2773	1318	2053	509496	246891	-230	756847	2116725	0.424	0.709
Troll	N.W.	30	38	45	5271	6359	5810	725176	891156	-808	1617947	1183089	0.219	0.187
Troll	N.W.	35	20	8	2942	1199	2078	429458	178385	-153	608149	3038769	0.375	0.839
Troll	S.W.	27	1	0	163	0	82	26419	0	0	26419	26612	1.974	1.982
Troll	S.W.	28	1	3	119	364	241	14095	43855	-16	57982	60113	1.000	1.019
Troll	S.W.	29	3	3	354	361	358	41447	43019	-51	84568	44	0.813	0.019
Troll	S.W.	30	5	6	634	775	704	79628	99151	-121	179020	19881	0.601	0.200
Troll	S.W.	35	9	11	1325	1650	1486	193352	245175	-428	439384	105425	0.446	0.219
Troll	N.E.	28	1	1	90	92	91	8019	8323	-2	16345	3	1.407	0.019
Troll	N.E.	30	5	5	671	683	677	89081	92457	-182	181903	157	0.630	0.019
Troll	N.E.	35	0	1	0	204	101	0	41570	0	41570	41795	2.013	2.019
Troll	S.E.	28	0	2	0	219	109	0	23572	0	23572	48063	1.414	2.019
Troll	S.E.	29	1	5	68	348	207	4595	23691	-39	28364	78271	0.814	1.352
Troll	S.E.	30	5	16	434	1415	920	37217	123206	-184	160791	962851	0.436	1.066
Troll	S.E.	35	9	7	1635	1296	1467	292526	236661	-2324	533835	115321	0.498	0.231
Average											1316196	1462371	0.862	0.789
Total					24005	19161	21606	15484633	6871255					
95% Confidence Intervals					+/-7713	+/-5138								

Appendix E Table 7. Estimated contribution of coho salmon release represented by tag codes 042257 and 042258 to the commercial net and trap fisheries and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1984. Of the total release of 328,559 coho salmon, 10,349 (or 3.15%) possessed tag code 042257 and 10,159 (or 3.09%) possessed tag code 042258. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	Quadrant	Stat. Week	Number of Tags Recovered 042257	Number of Tags Recovered 042258	Estimated Contribution 042257	Estimated Contribution 042258	Estimated Contribution Combined	Variance of Estimated Contribution 042257	Variance of Estimated Contribution 042258	Covariance	Variance of Difference MCBH	Variance of Difference Empirical	Coefficient of Variance MCBH	Coefficient of Variance Empirical
Seine	101	30	2	0	204	0	103	20559	0	0	20559	41602	1.393	1.982
Seine	101	32	0	1	0	200	99	0	39329	0	39329	40061	2.000	2.019
Seine	101	34	11	7	1549	1004	1279	215968	142787	-393	359540	296840	0.469	0.426
Seine	101	35	15	16	1474	1602	1538	142642	157866	-884	302276	16310	0.358	0.083
Seine	101	36	3	2	265	180	223	23037	15955	-41	39074	7216	0.888	0.382
Seine	101	37	1	5	201	1024	609	40100	205721	-581	246984	677225	0.816	1.352
Seine	101	38	1	4	331	1350	836	107042	415293	-9511	541356	1037156	0.880	1.219
Seine	102	33	0	1	0	355	176	0	123626	0	123626	126057	1.999	2.019
Seine	102	35	1	6	237	1451	838	55521	328116	-3458	390554	1471961	0.745	1.447
Seine	102	36	4	2	149	76	113	5391	2802	-7	8207	5348	0.803	0.648
Seine	103	34	0	1	0	402	199	0	160637	0	160637	161502	2.013	2.019
Seine	104	29	1	1	142	144	143	19806	20557	-90	40544	7	1.409	0.019
Seine	104	30	1	0	205	0	103	41673	0	0	41673	41890	1.977	1.982
Seine	104	31	1	0	171	0	86	29051	0	0	29051	29309	1.973	1.982
Seine	104	32	4	2	818	417	619	165965	86245	-239	252687	161053	0.812	0.648
Seine	104	33	2	3	248	379	313	30449	47393	-22	77885	17127	0.893	0.419
Seine	104	35	2	2	306	312	309	46516	48278	-99	94991	33	0.997	0.019
Seine	105	35	1	0	63	0	32	3910	0	0	3910	3978	1.965	1.982
Seine	106	35	3	1	389	132	262	48802	17163	-411	66786	65945	0.988	0.982
Gillnet	101	27	1	0	87	0	44	7265	0	0	7265	7563	1.942	1.982
Gillnet	101	29	0	1	0	78	39	0	5347	0	5347	6095	1.891	2.019
Gillnet	101	31	0	1	0	88	44	0	7163	0	7163	7732	1.943	2.019
Gillnet	101	32	1	0	71	0	36	5014	0	0	5014	5086	1.968	1.982
Gillnet	101	34	4	1	425	108	268	44594	11580	-11	56195	100126	0.885	1.182
Gillnet	101	36	5	8	275	447	360	14778	24527	-30	39364	29899	0.551	0.480
Gillnet	101	37	8	4	528	269	400	34239	17791	-40	52109	67106	0.571	0.648
Gillnet	101	38	8	11	448	627	536	24568	35053	-40	59700	32171	0.455	0.334
Gillnet	106	27	0	1	0	102	51	0	10307	0	10307	10420	2.008	2.019
Gillnet	106	28	0	1	0	106	53	0	11138	0	11138	11248	2.009	2.019
Gillnet	106	29	0	1	0	111	55	0	12276	0	12276	12391	2.009	2.019
Gillnet	106	33	2	0	129	0	65	8244	0	0	8244	16751	1.390	1.982
Gillnet	106	34	0	1	0	82	41	0	6686	0	6686	6772	2.006	2.019
Gillnet	106	35	8	7	585	522	554	42165	38305	-62	80594	4043	0.513	0.115
Gillnet	106	36	1	4	94	381	236	8665	35934	-13	44626	82837	0.894	1.219
Gillnet	106	37	4	1	333	85	210	27210	7085	-31	34358	61475	0.883	1.182
Fishtrap	101	35	2	1	87	44	66	3691	1919	-6	5622	1825	1.138	0.648
Average											91269	129560	1.290	1.264
Total					9814	12078	10935	1216865	2036879					
95% Confidence Intervals					+/-2162	+/-2797								

Appendix E Table 8. Estimated contribution of coho salmon release represented by tag codes 042259 and 042260 to the commercial troll fishery and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1984. Of the total release of 629,279 coho salmon, 10,774 (or 1.71%) possessed tag code 042259 and 10,630 (or 1.69%) possessed tag code 042260. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	Quadrant	Stat. Week	Number of Tags Recovered 042259	042260	Estimated Contribution 042259	042260	Combined	Variance of Estimated Contribution 042259	042260	Covariance	Variance of Difference MCBH	Empirical	Coefficient of Variance MCBH	Empirical
Troll	N.W.	27	1	2	3801	7705	5740	14436790	29649830	-11285	44109190	15239550	1.157	0.680
Troll	N.W.	28	3	1	1047	354	703	364279	124782	-57	489175	480747	0.995	0.987
Troll	N.W.	29	7	6	2381	2068	2226	807137	710775	-310	1518532	97653	0.554	0.140
Troll	N.W.	30	9	15	2297	3880	3083	583648	999186	-215	1583263	2505764	0.408	0.513
Troll	N.W.	35	4	3	1082	823	954	291818	224854	-39	516750	67406	0.754	0.272
Troll	S.W.	27	2	0	600	0	302	179137	0	0	179137	360284	1.401	1.987
Troll	S.W.	28	5	1	1097	222	663	239093	49198	-90	288470	764812	0.811	1.320
Troll	S.W.	29	0	1	0	220	109	0	48265	0	48265	48504	2.009	2.014
Troll	S.W.	30	2	3	466	709	587	108274	166809	-81	275245	58908	0.894	0.413
Troll	S.W.	35	4	4	1083	1098	1091	292047	300027	-233	592540	215	0.706	0.013
Troll	N.E.	27	1	0	366	0	184	133615	0	0	133615	134087	1.983	1.987
Troll	N.E.	29	0	1	0	212	106	0	44905	0	44905	45128	2.009	2.014
Troll	N.E.	30	1	1	247	250	248	60604	62260	-25	122914	11	1.411	0.013
Troll	S.E.	28	1	1	198	201	199	38795	39856	-199	79049	7	1.411	0.013
Troll	S.E.	29	1	4	126	510	316	15658	64028	-105	79896	147470	0.893	1.214
Troll	S.E.	30	3	7	479	1134	804	76027	182030	-163	258382	427990	0.632	0.813
Troll	S.E.	35	4	3	1337	1016	1178	443606	342171	-1491	788759	102829	0.754	0.272
Average											3006358	1204786	1.105	0.863
Total					16607	20402	18492	18070528	33008976					
95% Confidence Intervals					+/-8332	+/-11261								

Appendix E Table 9. Estimated contribution of coho salmon release represented by tag codes 042259 and 042260 to the commercial net and trap fisheries and comparison of the associated variance. Tag fish were released from the Whitman Lake hatchery in 1984. Of the total release of 629,279 coho salmon, 10,774 (or 1.71%) possessed tag code 042259 and 10,630 (or 1.69%) possessed tag code 042260. Empirical variances are the square of the difference between the unrounded estimated contributions of each tag code and are different than the square of the rounded difference.

Fishery	Quadrant	Stat. Week	Number of Tags Recovered		Estimated Contribution			Variance of Estimated Contribution		Covariance	Variance of Difference		Coefficient of Variance	
			042259	042260	042259	042260	Combined	042259	042260		MCBH	Empirical	MCBH	Empirical
Seine	101	32	1	2	361	733	546	128557	260556	-3533	396179	137832	1.153	0.680
Seine	101	34	3	7	777	1838	1304	200411	479918	-361	681050	1125331	0.633	0.813
Seine	101	35	4	7	723	1283	1001	129887	233256	-347	363838	313214	0.602	0.559
Seine	101	36	1	3	162	494	327	26172	80523	-69	106834	109755	1.000	1.014
Seine	101	37	1	1	370	375	372	136031	139747	-392	276561	25	1.412	0.013
Seine	101	38	1	1	609	618	613	362791	372695	-8007	751500	68	1.413	0.013
Seine	102	32	0	1	0	501	249	0	250495	0	250495	251200	2.011	2.014
Seine	102	33	1	0	641	0	323	403707	0	0	403707	411125	1.969	1.987
Seine	102	35	0	1	0	443	220	0	193420	0	193420	195825	2.001	2.014
Seine	102	36	1	6	69	417	242	4628	28438	-18	33101	121325	0.753	1.442
Seine	104	30	0	1	0	382	190	0	145223	0	145223	145646	2.011	2.014
Seine	104	31	1	0	315	0	159	98588	0	0	98588	99200	1.981	1.987
Seine	104	32	1	2	376	763	568	141043	289586	-201	431031	149290	1.156	0.680
Seine	104	33	1	1	228	231	230	51731	53145	-12	104901	10	1.411	0.014
Seine	104	35	0	2	0	571	284	0	162223	0	162223	326261	1.420	2.014
Seine	106	35	1	0	238	0	120	56166	0	0	56166	56857	1.975	1.987
Gillnet	101	28	1	0	126	0	63	15620	0	0	15620	15752	1.978	1.987
Gillnet	101	31	1	0	159	0	80	23482	0	0	23482	25219	1.917	1.987
Gillnet	101	35	0	1	0	137	68	0	18716	0	18716	18864	2.006	2.014
Gillnet	101	36	1	1	101	102	102	10098	10375	-3	20477	2	1.407	0.014
Gillnet	101	37	1	1	121	123	122	14616	15016	-4	29641	3	1.409	0.014
Gillnet	101	38	3	6	309	626	466	31464	64625	-28	96143	100595	0.665	0.680
Gillnet	106	33	2	1	238	121	180	28102	14438	-4	42547	13792	1.147	0.653
Gillnet	106	34	1	0	149	0	75	21926	0	0	21926	22086	1.979	1.987
Gillnet	106	35	6	2	808	273	542	107755	36932	-45	144776	285938	0.702	0.987
Gillnet	106	36	1	1	172	175	173	29471	30277	-11	59771	5	1.410	0.013
Fishtrap	101	35	0	1	0	81	40	0	6497	0	6497	6588	2.000	2.014
Average											182756	145623	1.464	1.170
Total					7052	10287	8658	2022246	2886101					
95% Confidence Intervals					+/-2787	+/-3330								

APPENDIX F

When cost is a consideration in planning a CWT project, the most accurate and precise estimate of n_1 is obtained when its variance is minimized within the constraints of available funds. In a CWT project, funds are spent mostly on tagging a portion of the smolts in a release and on checking commercial catches for marked fish (missing adipose fins). These activities result in numbers tagged (r_t) and numbers checked (n_2), respectively. Each of these activities have a unit cost (C_{r_t} and C_{n_2}) that together with tagging and checking levels produce a total cost, C_T:

[F1]

$$C_T = C_{r_t} r_t + C_{n_2} n_2$$

Most precision is obtained at values of r_t and n_2 that minimize the variance, Equation 13, and make Equation F1 true. Lagrangian multilpliers and differentiation are used to find these values for r_t and n_2. Note that Θ is defined in terms of r_t such that $\Theta = r_t/R$ where R is the total number of tagged and untagged fish in the release.

First, Equation F1 is rearranged and both sides multiplied by the dummy variable λ:

[F2]

$$\lambda C_{r_t} r_t + \lambda C_{n_2} n_2 - \lambda C_T = 0$$

Equation F2 is added to the variance, Equation 40:

[F3]

$$\mathrm{Var}(\hat{n}_1) = \frac{m_1(m_2-1)a_1(a_2-1)n_1(n_1-1)}{m_2(m_1-1)a_2(a_1-1)} + \frac{m_1 a_1 N n_1}{m_2 a_2 n_2 \Theta} - n_1^2 + \lambda C_{r_t} r_t + \lambda C_{n_2} n_2 - \lambda C_T$$

The partial derivatives of Equation F3 with respect to r_t and n_2 are now taken and set to zero:

$$\frac{\partial Var(\hat{n}_1)}{\partial n_2} = - \frac{m_1 a_1 N n_1}{m_2 a_2 n_2^2 \Theta} + \lambda C_{n_2} = 0 \qquad [F4]$$

$$\frac{\partial Var(\hat{n}_1)}{\partial r_t} = - \frac{m_1 a_1 N n_1}{m_2 a_2 n_2 r_t \Theta} + \lambda C_{r_t} = 0 \qquad [F5]$$

since $\partial\Theta^{-1}/\partial r_t$ is $-(r_t\Theta)^{-1}$. Note that the second derivatives of Equation F3 with respect to r_t and n_2 will always be positive which shows that the solution of simultaneous Equations F4-F5 for values of these two variables will produce a minimum variance. Equation F4 is used to solve for λ:

$$\lambda = \frac{m_1 a_1 N n_1}{m_2 a_2 n_2^2 \Theta C_{n_2}} \qquad [F6]$$

The solution for λ in Equation F6 is put into Equation F5, and the latter equation is reduced to:

$$C_{n_2} n_2 = C_{r_t} r_t \qquad [F7]$$

which results in Equation F1 being:

$$C_{n_2} n_2 = C_{r_t} r_t = C_T/2 \qquad [F8]$$

From Equation F8 we find that the variance is minimal when half the available money is allocated to the tagging program and half to the sampling program. Note that Equation F8 was derived under the conditions of a single release and a single sampling stratum.

The same procedure is used to minimize the variance under conditions of one release returning to several sampling strata. The cost function under these conditions is:

[F9]

$$C_T = \sum_{i=1}^{s} C_{n_{2i}} n_{2i} + C_{r_t} r_t$$

for s sampling strata. Assuming that sampling activities are independent among strata, the covariance term of text Equation 17 is zero for all i and Equation 17 reduces to:

[F10]

$$Var(TC_{n_1}) = \sum_{i=1}^{s} Var(\hat{n}_1)_i$$

All elements in Equation F9 are multiplied by λ and the resulting equation is added to Equation F10:

[F11]

$$Var(TC_{n_1}) = \sum_{i=1}^{s} Var(\hat{n}_1)_i + \sum_{i=1}^{s} \lambda C_{n_{2i}} n_{2i} + \lambda C_{r_t} r_t - \lambda C_T$$

The partial derivatives of Equation F11 with respect to each of the n_{2i} and r_t are taken and set equal to zero, resulting in s + 1 equations:

[F12]

$$\frac{\partial Var(TC_{n_1})}{\partial n_{2i}} = - \frac{m_{1i} a_{1i} N_i n_{1i}}{m_{2i} a_{2i} n_{2i}{}^2 \Theta} + \lambda C_{n_{2i}} = 0$$

for each i strata (i = 1 to s), and

[F13]

$$\frac{\partial Var(TC_{n_1})}{\partial r_t} = - \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1i}}{m_{2i} a_{2i} n_{2i} r_t \Theta} + \lambda C_{r_t} = 0$$

Equation F12 is divided by n_{2i}, Equation F13 is divided by r_t and both are rearranged:

[F14]

$$\lambda C_{n_{2i}} n_{2i} = \frac{m_{1i} a_{1i} N_i n_{1i}}{m_{2i} a_{2i} n_{2i} \Theta}$$

for each i strata (i = 1 to s), and

[F15]

$$\lambda C_{r_t} r_t = \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1i}}{m_{2i} a_{2i} n_{2i} \Theta}$$

The s equations represented by Equation 14 are relevant for all i strata. Therefore, Equation 14 is summed over all i strata:

[F16]

$$\sum_{i=1}^{s} \lambda C_{n_{2i}} n_{2i} = \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1i}}{m_{2i} a_{2i} n_{2i} \Theta}$$

The righthand sides of Equations F15 and F16 are equivalent. Substitution and cancelling the λ results in equations analogous to Equations F7 and F8:

[F17]

$$\sum_{i=1}^{s} C_{n_{2i}} n_{2i} = C_{r_t} r_t$$

[F18]

$$\sum_{i=1}^{s} C_{n_{2i}} n_{2i} = C_{r_t} r_t = C_T/2$$

Regardless of the level of stratification of the sampling program, for a given amount of funding, the variance of the estimated contribution is minimal when half the money is allocated to the tagging program and half the money is allocated to the sampling program.

If a minimum variance of the total contribution estimate of several releases which are harvested in several catch strata is desired, the constraint for a given level of funding is:

[F19]

$$C_T = \sum_{i=1}^{s} C_{n_{2i}} n_{2i} + \sum_{j=1}^{t} C_{r_{tj}} r_{tj}$$

for i = 1 to s sampling strata and j = 1 to t releases. Text Equation 17 defines the variance of the total estimated contribution of all releases across all sampling strata:

[F20]

$$Var(TC_{n_1}) = \sum_{i=1}^{s} \sum_{j=1}^{t} Var(\hat{n}_1)_{ij} + 2 \sum_{i=1}^{s} \sum_{j=1}^{t} \sum_{k>j}^{t} Cov(\hat{n}_{1j};\hat{n}_{1k})_i$$

Under the approximation of $\left(\frac{N}{N-1}\right)\left(\frac{n_2}{n_2-1}\right) \cong 1$, substitution of expressions for variance (Equation 38) and covariance (Equation D24), and rearranging the terms results in:

[F21]

$$\mathrm{Var}(TC_{n_1}) = \sum_{i=1}^{s} \sum_{j=1}^{t} \left(\frac{m_{1i}(m_{2i}-1)a_{1i}(a_{2i}-1)n_{1ij}(n_{1ij}-1)}{m_{2i}(m_{1i}-1)a_{2i}(a_{1i}-1)} + \frac{m_{1i}a_{1i}N_i n_{1ij}}{m_{2i}a_{2i}n_{2i}\Theta_j} - n_{1ij}^{2} \right)$$

$$+ 2 \sum_{i=1}^{s} \sum_{j=1}^{t} \sum_{k>j}^{t} \left(\frac{m_{1i}(m_{2i}-1)a_{1i}(a_{2i}-1)n_{1ij}n_{1ik}}{m_{2i}(m_{1i}-1)a_{2i}(a_{1i}-1)} - n_{1ij}n_{1ik} \right)$$

All elements in the constraint are multiplied by λ and the resulting equation is added to Equation F21. The partial derivatives of the resulting equation with respect to n_{2i} and r_{tj} are set equal to zero:

[F22]

$$\frac{\partial \mathrm{Var}(TC_{n_1})}{\partial n_{2i}} = - \sum_{j=1}^{t} \frac{m_{1i}a_{1i}N_i n_{1ij}}{m_{2i}a_{2i}n_{2i}^{2}\Theta_j} + \lambda C_{n_2 i} = 0$$

for each i strata (i = 1 to s), and

[F23]

$$\frac{\partial \mathrm{Var}(TC_{n_1})}{\partial r_{tj}} = - \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i} r_{ti} \Theta_i} + \lambda C_{r_{tj}} = 0$$

for each j release (j = 1 to t). To simplify notation, define S_j as the sum of the all the terms under the summation sign except r_{tj} in Equation F23 for any given value of j:

[F24]

$$S_j = \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i} \Theta_j}$$

By substituting S_j into Equation 23 and solving for λ, we get:

[F25]

$$\lambda = \frac{S_j}{C_{r_t j} r_{tj}}$$

For any given i and for any given j, Equation F25 can be substituted into Equation F22 for λ, resulting in:

[F26]

$$\frac{S_j C_{n_2 i}}{C_{r_t j} r_{tj}} = \sum_{j=1}^{t} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i}{}^2 \Theta_j}$$

Equation F26 is multiplied by n_{2i} and terms are rearranged to give:

[F27]

$$S_j C_{n_{2i}} n_{2i} = C_{r_t j} r_{tj} \sum_{j=1}^{t} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i} \Theta_j}$$

Because Equation F27 is relevant for any stratum i, the equality can be summed over all strata (i = 1 to s). Rearrangement of the summations gives:

[F28]

$$S_j \sum_{i=1}^{s} C_{n_{2i}} n_{2i} = C_{r_t j} r_{tj} \sum_{j=1}^{t} \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i} \Theta_j}$$

The rightmost summation in Equation 28 is defined in Equation 24. With substitution of Equation 24, Equation 28 is identical to:

[F29]

$$S_j \sum_{i=1}^{s} C_{n_{2i}} n_{2i} = C_{r_t j} r_{tj} \sum_{j=1}^{t} S_j$$

Equation F29 is relevant for any release j, and therefore the equality can be summed over all releases:

[F30]

$$\sum_{j=1}^{t} S_j \sum_{i=1}^{s} C_{n_2 i} n_{2i} = \sum_{j=1}^{t} C_{r_t j} r_{tj} \sum_{j=1}^{t} S_j$$

By cancelling the $\sum_{j=1}^{t} S_j$ from both sides,
Equation 30 reduces to

[F31]

$$\sum_{i=1}^{s} C_{n_2 i} n_{2i} = \sum_{j=1}^{t} C_{r_t j} r_{tj}$$

By substitution by into Equation F19, it can be again be shown that the optimum allocation of available funds is achieved when one-half of the amount is given to the tagging program and one-half is given to the sampling program.

[F32]

$$\sum_{i=1}^{s} C_{n_2 i} n_{2i} = \sum_{j=1}^{t} C_{r_t j} r_{tj} = C_T/2$$

APPENDIX G

In sampling and tagging programs, the minimum variance of the total estimated contribution is obtained when fiscal resources are allocated equally to each program. This is true if multiple releases are to be tagged, if several strata are to be sampled, or if both the tagging and sampling programs are stratified (see Appendix E). Mathematically, this relationship was presented in Equation E32:

$$\sum_{i=1}^{s} C_{n_{2i}} n_{2i} = \frac{C_T}{2} = F_{n_2} \qquad [G1]$$

$$\sum_{j=1}^{t} C_{r_{tj}} r_{tj} = \frac{C_T}{2} = F_{r_t} \qquad [G2]$$

Where F_{n_2} and F_{r_t} are the levels of funding allocated to the sampling and tagging program, respectively, which will minimize the variance. Equation 22 defines the variance for catches from several releases estimated with a stratified sampling program. Variance formulas for situations with one release and many sampling strata or with one sampling stratum and many releases are special cases of Equation 22. Equation G1 is the cost function for the catch sampling program and Equation G2 is the cost function for the tagging program.

In order to obtain an equation which prescribes optimal allocation of effort among strata within the catch sampling program, Langrangian multipliers (λ) and differentiation are used. Equation G1 is multiplied by λ and added to the variance function, Equation 22:

[G3]

$$\mathrm{Var}(TC_{n_1}) = \sum_{i=1}^{s}\sum_{j=1}^{t} \mathrm{Var}(\hat{n}_1)_{ij} + 2\sum_{i=1}^{s}\sum_{j=1}^{t}\sum_{k>j}^{t} \mathrm{Cov}(\hat{n}_{1j};\hat{n}_{1k})_i + \sum_{i=1}^{s}\lambda C_{n_2 i} n_{2i} - \frac{\lambda C_T}{2}$$

The partial derivatives of Equation E3 with respect to r_t and n_2 are now taken and set to zero:

[G4]

$$\frac{\partial \mathrm{Var}(TC_{n_1})}{\partial n_{2i}} = -\sum_{j=1}^{t} \frac{m_{1i} a_{1i} N_i n_{1i}}{m_{2i} a_{2i} n_{2i}^{2} \Theta} + \lambda C_{n_2 i} = 0$$

Solving Equation G4 in terms of stratum i and a second stratum h results in:

[G5]

$$\lambda = \sum_{j=1}^{t} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i}^{2} \Theta_j C_{n_2 i}}$$

[G6]

$$\lambda = \sum_{j=1}^{t} \frac{m_{1h} a_{1h} N_h n_{1hj}}{m_{2h} a_{2h} n_{2h}^{2} \Theta_j C_{n_2 h}}$$

Noting that the righthand sides of Equations G5 and G6 are equal, n_{2i} can be solved for:

[G7]

$$n_{2i} = n_{2h}\left[\frac{m_{2h}a_{2h}m_{1i}a_{1i}N_iC_{n_2h}\sum_{j=1}^{t}\frac{n_{1ij}}{\Theta_j}}{m_{2i}a_{2i}m_{1h}a_{1h}N_hC_{n_2i}\sum_{j=1}^{t}\frac{n_{1hj}}{\Theta_j}}\right]^{\frac{1}{2}}$$

Removing C_{n_2i} and C_{n_2h} from under the radical and rearranging results in:

[G8]

$$C_{n_2i}n_{2i} = C_{n_2h}n_{2h}\left[\frac{m_{2h}a_{2h}m_{1i}a_{1i}N_iC_{n_2i}\sum_{j=1}^{t}\frac{n_{1ij}}{\Theta_j}}{m_{2i}a_{2i}m_{1h}a_{1h}N_hC_{n_2h}\sum_{j=1}^{t}\frac{n_{1hj}}{\Theta_j}}\right]^{\frac{1}{2}}$$

Since the product of the number of fish sampled in a given stratum i and the cost of sampling each individual fish in the given stratum i is the cost of sampling the stratum (F_{n_2i}), the ratio of the cost of sampling one stratum to the cost of sampling another stratum is shown to be:

[G9]

$$\frac{F_{n_2i}}{F_{n_2h}} = \left[\frac{m_{2h}a_{2h}m_{1i}a_{1i}N_iC_{n_2i}\sum_{j=1}^{t}\frac{n_{1ij}}{\Theta_j}}{m_{2i}a_{2i}m_{1h}a_{1h}N_hC_{n_2h}\sum_{j=1}^{t}\frac{n_{1hj}}{\Theta_j}}\right]^{\frac{1}{2}}$$

Equation G9 is the generalized formula for calculating the relative funding for strata i and h that will produce a minimum variance in contribution estimates from a catch sampling program with two or more releases contributing to the catch. For the special case when there are multiple catch sampling strata but only one release of interest, Equation G9 will reduce to:

[G10]

$$\frac{F_{n_2 i}}{F_{n_2 h}} = \left[\frac{m_{2h} a_{2h} m_{1i} a_{1i} N_i C_{n_2 i} n_{1ij}}{m_{2i} a_{2i} m_{1h} a_{1h} N_h C_{n_2 h} n_{1hj}} \right]^{\frac{1}{2}}$$

Optimum allocation within the tagging program can be estimated by minimizing Equation 17 subject to the constraint of Equation G2. Using Lagrangian multipliers (λ) and adding Equation G2 to Equation 17 results in:

[G11]

$$\mathrm{Var}(TC_{n_1}) = \sum_{i=1}^{s} \sum_{j=1}^{t} \mathrm{Var}(\hat{n}_1)_{ij} + 2 \sum_{i=1}^{s} \sum_{j=1}^{t} \sum_{k>j}^{t} \mathrm{Cov}(\hat{n}_{1j};\hat{n}_{1k})_i + \sum_{j=1}^{t} \lambda C_{r_t j} r_{tj} - \lambda F_{r_t}$$

The partial derivative of Equation G11 is taken with respect to r_{tj} and set equal to zero:

[G12]

$$\frac{\partial \mathrm{Var}(TC_{n_1})}{\partial r_{tj}} = - \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i} r_{tj} \Theta_j} + \lambda C_{r_t j} = 0$$

Note that $\theta = r_{tj}/R_j$ and $\frac{\partial\Theta_j^{-1}}{\partial r_{tj}} = -(r_{tj}\Theta_j)^{-1}$.

Substituting and solving for λ in terms of release j and in terms of release 1 gives:

[G13]

$$\lambda = \frac{R_j}{C_{r_t j} r_{tj}^2} \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i}}$$

[G14]

$$\lambda = \frac{R_1}{C_{r_t 1} r_{t1}^2} \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1i1}}{m_{2i} a_{2i} n_{2i}}$$

Since Equation G13 equals Equation G14, the two righthand sides are equated and r_{tj} is solved for:

[G15]

$$r_{tj} = r_{t1} \left(\frac{C_{r_t 1} R_j \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i}}}{C_{r_t j} R_1 \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1i1}}{m_{2i} a_{2i} n_{2i}}} \right)^{\frac{1}{2}}$$

Multiplying both sides of Equation G15 by $C_{r_t j}$ and rearranging terms gives:

[G16]

$$C_{r_t j} r_{tj} = C_{r_t 1} r_{t1} \left(\frac{C_{r_t j} R_j \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i}}}{C_{r_t 1} R_1 \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1i1}}{m_{2i} a_{2i} n_{2i}}} \right)^{\frac{1}{2}}$$

Since the product of the number of fish tagged i a given release j and the cost of tagging each individual fish in the release j is the cost of tagging the release ($F_{r_t j}$), the ratio of the cost of one release to the cost of tagging another release is shown to be:

[G17]

$$\frac{F_{r_t j}}{F_{r_t 1}} = \left(\frac{C_{r_t j} R_j \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1ij}}{m_{2i} a_{2i} n_{2i}}}{C_{r_t 1} R_1 \sum_{i=1}^{s} \frac{m_{1i} a_{1i} N_i n_{1i1}}{m_{2i} a_{2i} n_{2i}}} \right)^{\frac{1}{2}}$$

For the special case when there are several releases in only one catch stratum, Equation G17 will reduce to

[G17]

$$\frac{F_{r_t j}}{F_{r_t 1}} = \left(\frac{C_{r_t j} R_j n_{1ij}}{C_{r_t 1} R_1 n_{1i1}} \right)^{\frac{1}{2}}$$